Pendular models for walking over rough terrains

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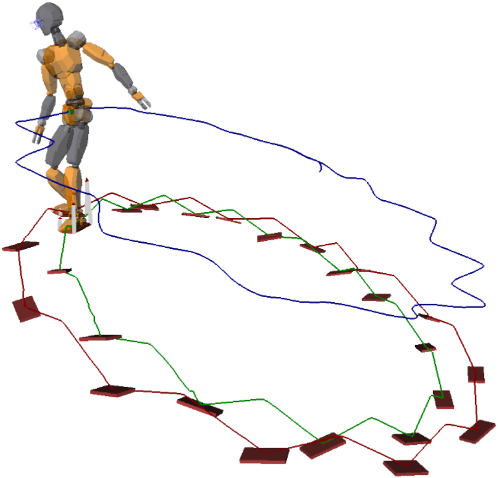
Goal

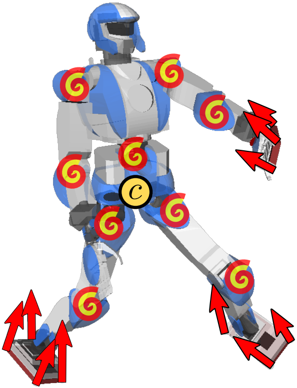


Standard model reduction

Multi-body systems

Equation of motion

$$M\ddot{q} + h(q, \dot{q}) = S^T \tau + J_c^T F$$



Constraints

- $\tau \in \{\text{feasible torques}\}$
- $F \in \{\text{feasible contact forces}\}$

Assumption

- (Rigid bodies)

Newton-Euler dynamics

Equations of motion

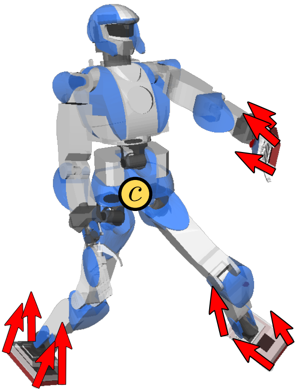
$$\begin{aligned} \ddot{c} &= \frac{1}{m} \sum_i f_i + \vec{g} \\ \dot{L}_c &= \sum_i (p_i - c) \times f_i \end{aligned}$$

Constraints

- Friction cones: $\forall i, f_i \in \mathcal{C}_i$

Assumption

- Infinite torques



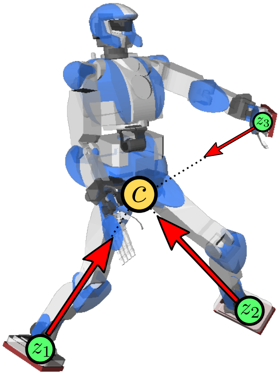
Angular momentum regulation

Pendular mode

$$\dot{L}_c = 0$$

Conserve the angular momentum at the center-of-mass

- **Pro:** enables exact forward integration
- **Con:** assumes $\dot{L}_c = 0$ feasible regardless of joint state



Linear Inverted Pendulum Mode

Equation of motion

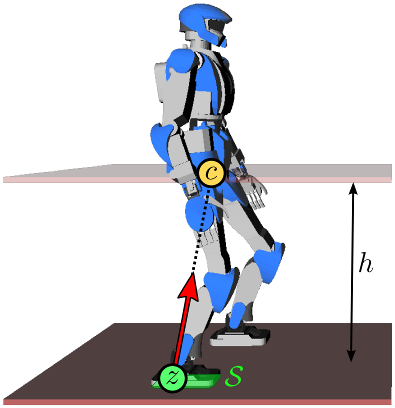
$$\ddot{c} = \omega^2 (c - z) + \vec{g}$$

Constraints

- ZMP support area: $z \in \mathcal{S}$

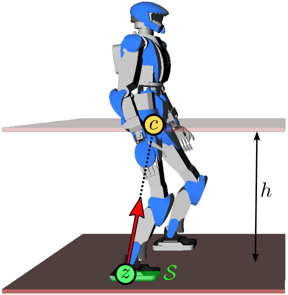
Assumptions

- Infinite torques
- Pendular mode $\dot{L}_c = 0$
- COM lies in a plane: $c_z = h$
- *Infinite friction*
- *Contacts are coplanar*



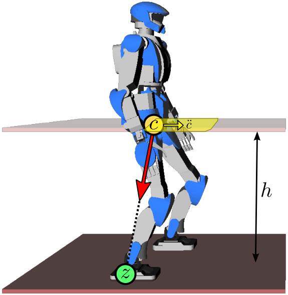
LIPM and CART-table

LIPM [Kaj+01]



- Input: $z \in \mathcal{S}$
- Output: $\ddot{c}$

CART-table [Kaj+03]



- Input: $\ddot{c} \in \omega^2(c - \mathcal{S}) + \vec{g}$
- Output: z

Polyhedral geometry: a tool for model reduction

Without infinite friction

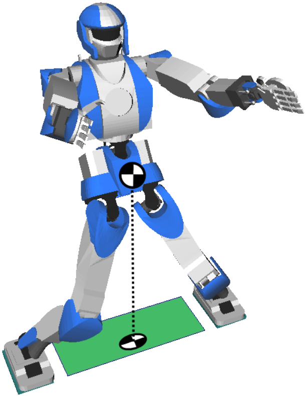
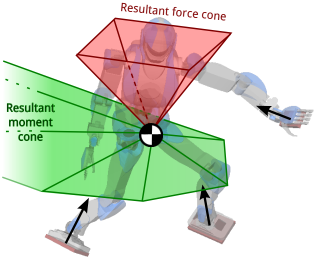


Figure : ZMP support area with friction [CPN17]

Polyhedral geometry

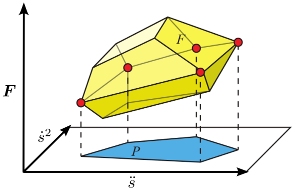
Geometric tool

- Apply linear maps to cones
- Project system constraints as support areas / volumes
- Construct *feasibility certificates* for reduced models



Algorithms and ressources

- Double description [FP96]
- Fourier-Motzkin elim. [Zie95]
- Polytope projection [JKM04]
- My website ;) [Car17]



How many contact points per contact?

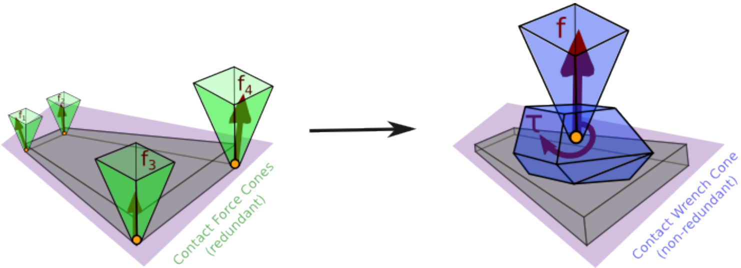


Figure : Reduce redundant friction cones into wrench cones [CPN15]

Torque-limited friction cones

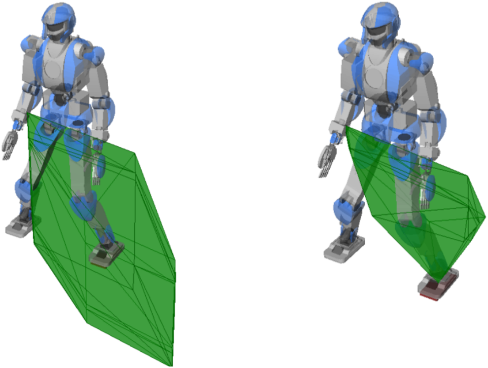


Figure : Friction cones that include actuation limits [Sam+17]

Without coplanar contacts

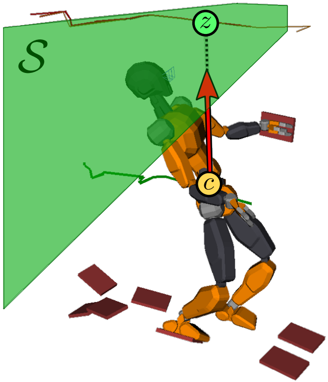
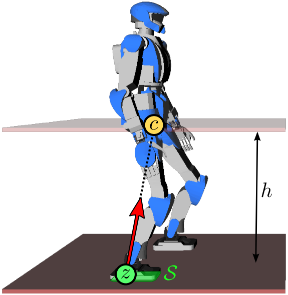


Figure : ZMP support area with non-coplanar contacts [CPN17]

From 2D to 3D locomotion

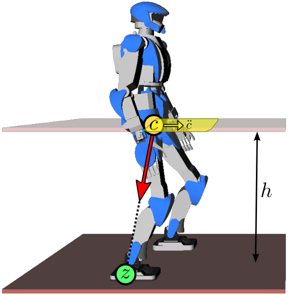
LIPM and CART-table

2D LIPM



- Input: $z \in \mathcal{S}$
- Output: $\ddot{c}$

2D CART-table



- Input: $\ddot{c} \in \omega^2(c - \mathcal{S}) + \vec{g}$
- Output: z

Linear Pendulum Mode

Equation of motion

$$\ddot{c} = \text{sign}(h)\,\omega^2(c - z) + \vec{g}$$

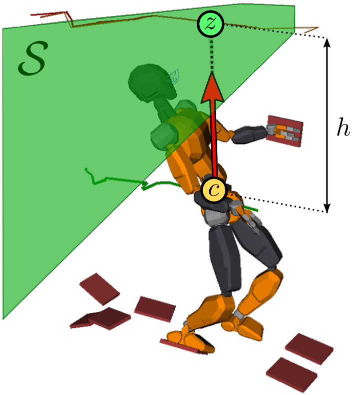
Constraints

- ZMP support area: $z \in \mathcal{S}$

Assumptions

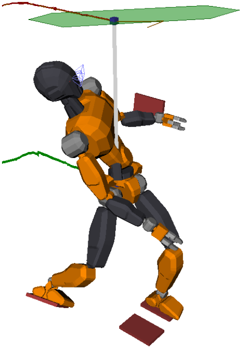
- Inf. torques & pendular mode
- COM and ZMP lie in parallel virtual planes distant by h

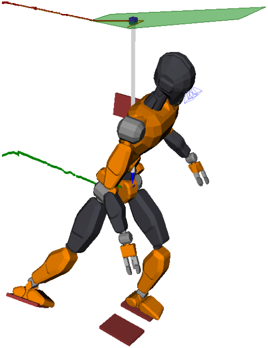
Note: COM is attractor or repulsor depending on sign(h)



Observation

ZMP support area $\mathcal{S}$ changes with COM position:





3D CART-table

COM acceleration cone

Algorithm [CK16]

Compute the 3D cone $\mathcal{C}$ of COM accelerations

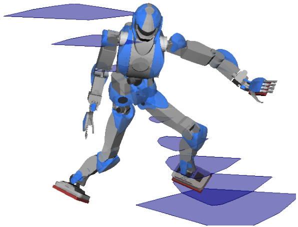


Figure : ZMP support areas for different values of $\pm\omega^2$

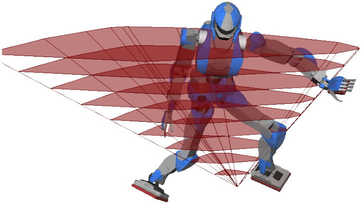
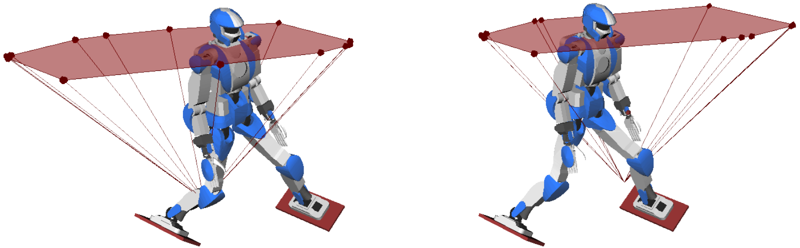


Figure : COM acceleration cone for the same stance

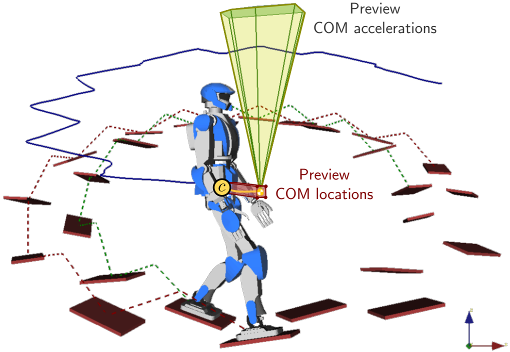
Observation

The cone $\mathcal{C}$ still depends on the COM position c:



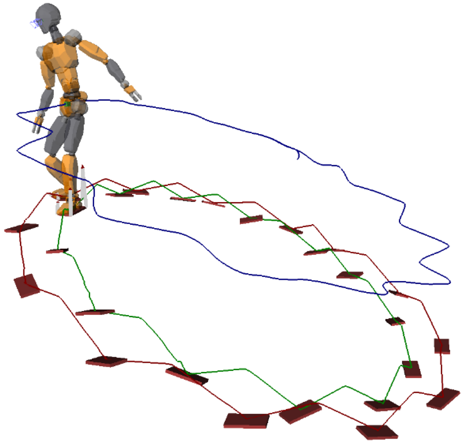
Predictive Control

Intersect cones $\mathcal{C}$ over all $c \in$ preview:



Walking patterns *not very dynamic*, but works surprisingly well!

Check it out!

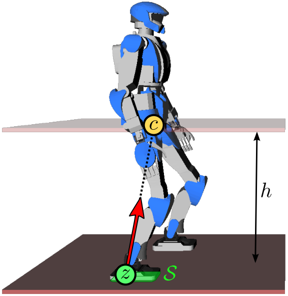


https://github.com/stephane-caron/3d-com-lmpc

3D Pendulum Mode

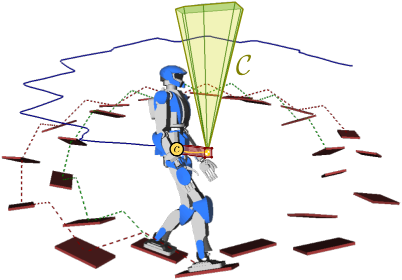
LIPM and CART-table

2D LIPM



- Input: $z \in \mathcal{S}$
- Output: $\ddot{c}$

3D COM-accel [CK16]



- Input: $\ddot{c} \in \mathcal{C}(c)$
- Output: z

Inverted Pendulum Mode

Linear Inverted Pendulum

$$\ddot{c} = \omega^2(c - z) + \vec{g}$$

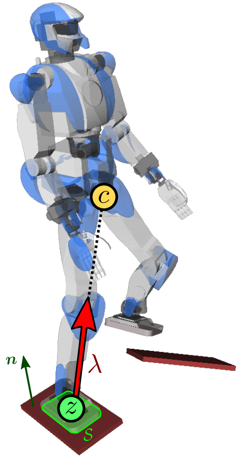
Plane assumption: $\omega = \sqrt{\frac{g}{h}}$

$$\downarrow$$

Remove this assumption:

Inverted Pendulum

$$\ddot{c} = \lambda(c - z) + \vec{g}$$



Inverted Pendulum Mode

Equation of motion

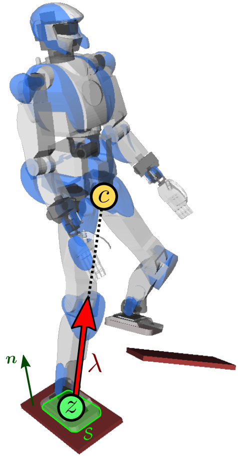
$$\ddot{c} = \lambda(c - z) + \vec{g}$$

Constraints

- Unilaterality $\lambda \geq 0$
- ZMP support area: $z \in \mathcal{S}$

Assumptions

- Infinite torques
- *Infinite friction*
- Pendular mode



Inverted Pendulum Mode with Friction

Equation of motion

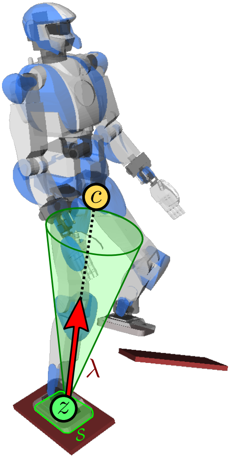
$$\ddot{c} = \lambda(c - z) + \vec{g}$$

Constraints

- Unilaterality $\lambda \geq 0$
- ZMP support area: $z \in \mathcal{S}$
- Friction: $c - z \in \mathcal{C}$

Assumptions

- Infinite torques
- Pendular mode

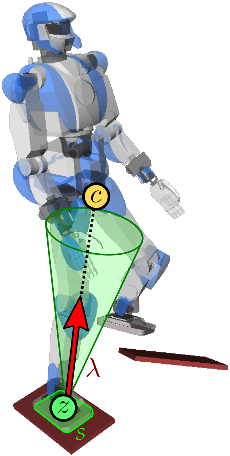


Inverted Pendulum Mode: Question

Equation of motion

$$\ddot{c} = \lambda(c - z) + \vec{g}$$

- Product bwn control and state
- Forward integration: how to make it **exact**?



Reformulation

Floating-base inverted pendulum (FIP)

Allow the ZMP to leave the contact area.[1]

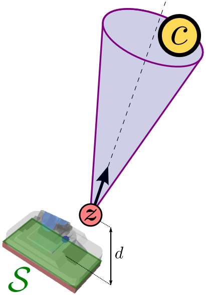


Figure : Friction constraint

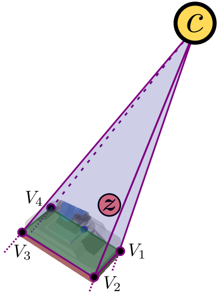


Figure : ZMP constraint

[1] At heart, it is used to locate the central axis of the contact wrench [SB04]

Floating-base Inverted Pendulum

Equation of motion

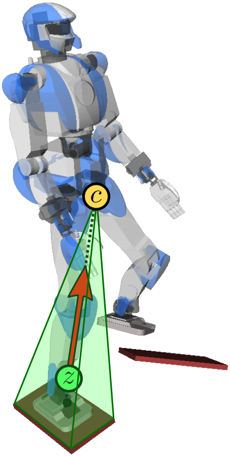
$$\ddot{c} = \omega^2 (c - z) + \vec{g}$$

Constraints [CK17]

- Friction: $c - z \in \mathcal{C}$
- ZMP support cone:
  $\forall i, e_i \cdot (v_i - c) \times (z - v_i) \leq 0$

Assumptions

- Infinite torques
- Pendular mode



Properties of FIP model

Equation of motion

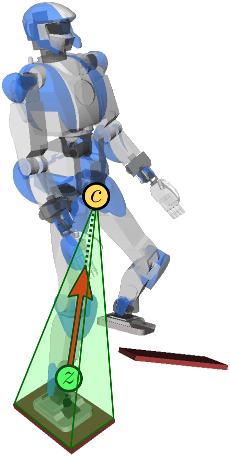
$$\ddot{c} = \omega^2 (c - z) + \vec{g}$$

- Forward integration is **exact**:

$$c(t) = \alpha_0 e^{\omega t} + \beta_0 e^{-\omega t} + \gamma_0$$

- Capture Point is defined:

$$\xi = c + \frac{\dot{c}}{\omega} + \frac{\vec{g}}{\omega^2}$$



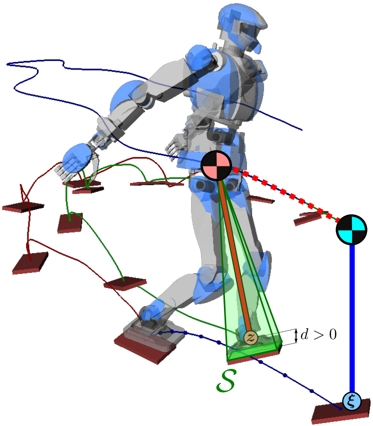
Model Predictive Control

NMPC Optimization

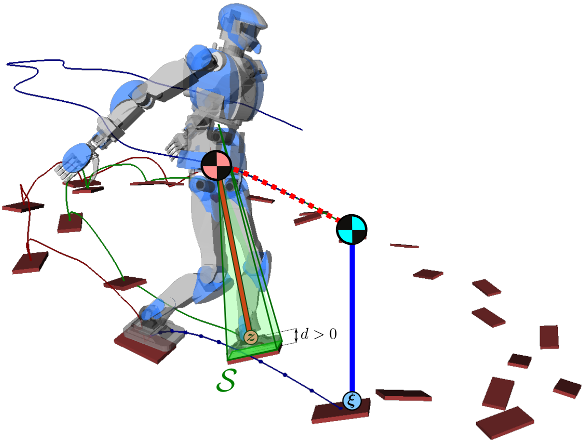
- Runs at 30 Hz
- Adapts step timings
- FIP for forward integration
- Sometimes fails...

Linear-Quadratic Regulator

- Runs at 300 Hz
- Takes over when NMPC fails



Check it out!

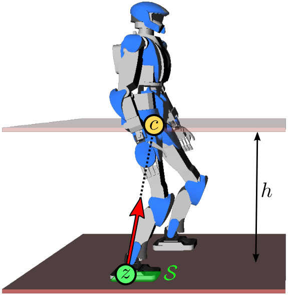


https://github.com/stephane-caron/dynamic-walking

Conclusion

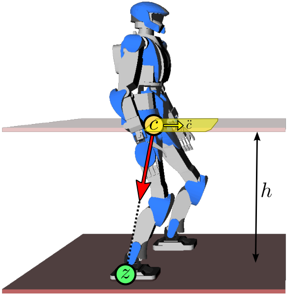
Conclusion

2D LIPM



- Input: $z \in \mathcal{S}$
- Output: $\ddot{c}$

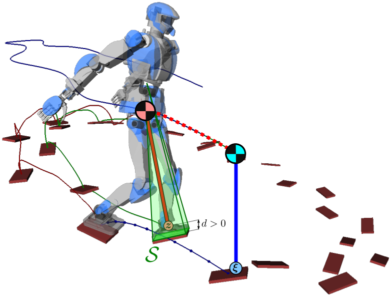
2D CART-table



- Input: $\ddot{c} \in \omega^2(c - \mathcal{S}) + \vec{g}$
- Output: z

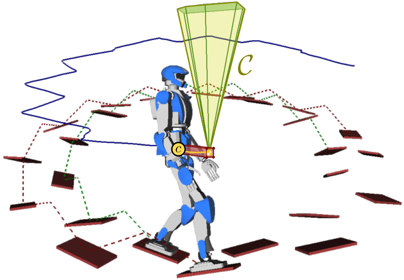
Conclusion

3D FIP [CK17]



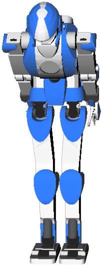
- Input: $z \in \mathcal{S}(c)$
- Output: $\ddot{c}$

3D COM-accel [CK16]



- Input: $\ddot{c} \in \mathcal{C}(c)$
- Output: z

Thank you for your attention!



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[Zie95] Günter M. Ziegler. *Lectures on polytopes*. Graduate texts in mathematics 152. New York: Springer-Verlag, 1995. 370 pp. ISBN: 978-0-387-94329-9 978-3-540-94329-7 978-0-387-94365-7 978-3-540-94365-5.