Reinforcement learning for legged robots

Stéphane Caron

November 16, 2023

Inria, École normale supérieure



RL in robotics

2020: Quadrupedal locomotion



Teacher-student residual reinforcement learning [Lee+20]

Video: https://youtu.be/oPNkeoGMvAE

2018: In-hand reorientation



LSTM policy with domain randomization [And+20]

Video: https://youtu.be/jwSbzNHGflM

2010: Helicopter stunts



Helicopter aerobatics through apprenticeship learning [ACN10]

Video: https://youtu.be/M-QUkgk3HyE

1997: Pendulum swing up

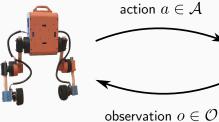


Swinging up an inverted pendulum from human demonstrations [AS97]

Video: https://youtu.be/g3I2VjeSQUM?t=294

Basics of reinforcement learning

Agent



reward $r \in \mathbb{R}$

Environment



Rewards

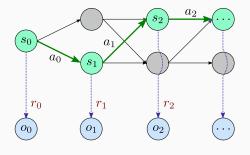






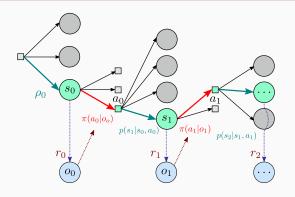
Image credit: L. M. Tenkes, source: https://araffin.github.io/post/sb3/

Partially observable Markov decision process (1/2)



- State: s_t , ground truth of the environment
- Action: a_t , decision of the agent (discrete or continuous)
- Observation: o_t , partial estimation of the state from sensors
- Reward: $r_t \in \mathbb{R}$, scalar feedback, often $r_t = r(s_t, a_t)$ or $r(s_t, a_t, s_{t+1})$

Partially observable Markov decision process (2/2)



	Deterministic	Stochastic	
Model:	$s_{t+1} = f(s_t, a_t)$	$s_{t+1} \sim p(\cdot s_t, a_t)$	how the environment evolves
Initial state:	s_0	$s_0 \sim \rho_0(\cdot)$	where we start from
Observation:	$o_t = h(s_t)$	$o_t \sim z(\cdot s_t)$	how sensors measure the world
Policy:	$a_t = g(s_t)$	$a_t \sim \pi(\cdot o_t)$	what the agent decides

Example: The Gymnasium API



```
import gymnasium as gym

with gym.make("CartPole-v1", render_mode="human") as env:
    env.reset()
    action = env.action_space.sample()
    for step in range(1000):
        observation, reward, terminated, truncated, _ = env.step(action)
        if terminated or truncated:
            observation, _ = env.reset()
        cart_position = observation[0]
        action = 0 if cart_position > 0.0 else 1
```

Same API for simulation and real robots



```
import gymnasium as gym

with gym.make("UpkieGroundVelocity-v1", frequency=200.0) as env:
    env.reset()
    action = 0.0 * env.action_space.sample()
    for step in range(1_000_000):
        observation, reward, terminated, truncated, _ = env.step(action)
        if terminated or truncated:
            observation, _ = env.reset()
        pitch = observation[0]
        action[0] = 10.0 * pitch # action is [ground_velocity]
```

Goal of reinforcement learning

Two last missing pieces:

- Episode: $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, \ldots)$ truncated or infinite¹
- Return: $R(\tau) = \sum_{t \in \tau} r_t$ or with discount $\gamma \in]0,1[:R(\tau) = \sum_{t \in \tau} \gamma^t r_t$

We can now state what reinforcement learning is about:

Goal of reinforcement learning

The goal of reinforcement learning is to find a policy that maximizes returns.

¹In practice episodes contain o_t rather than s_t . In RL, we implicitly assume that observations contain enough information to be in bijection with their corresponding states. See also Augmenting observations thereafter.

Stochastic reinforcement learning

In the stochastic setting, the goal of reinforcement learning is:

```
\max_{\pi} \mathbb{E}_{\tau}[R(\tau)]
s.t. \tau = (s_0, a_0, s_1, a_1, \ldots)
s_0 \sim \rho_0(\cdot)
o_0 \sim z(\cdot|s_0)
a_0 \sim \pi(\cdot|o_0)
s_1 \sim p(\cdot|s_0, a_0)
\vdots
```

Value functions (1/2)

State value functions *V*:

- On-policy: expected return from a given policy: $V^{\pi}(s) = \mathbb{E}_{\tau \sim \pi}(R(\tau)|s_0 = s)$
- Optimal: best return we can expect from a state: $V^*(s) = \max_{\pi} \mathbb{E}_{\tau \sim \pi}(R(\tau)|s_0 = s)$

State-action value functions Q:

- · On-policy: expected return from following policy:
 - $Q^{\pi}(s,a) = \mathbb{E}_{\tau \sim \pi}(R(\tau)|s_0 = s, a_0 = a)$
- Optimal: best return we can expect: $Q^*(s,a) = \max_{\pi} \mathbb{E}_{\tau \sim \pi}(R(\tau)|s_0 = s, a_0 = a)$

Value functions (2/2)

Value functions satisfy the Bellman equation:

Bellman equation

$$V^{*}(s) = \max_{\pi} \mathbb{E}_{a \sim \pi(\cdot|s), (r,s') \sim p(s'|s,a)} [r + \gamma V^{*}(s')]$$

This is a connection to optimal control (e.g. differential dynamic programming) and Q-learning, but not our topic today.

Components of an RL algorithm

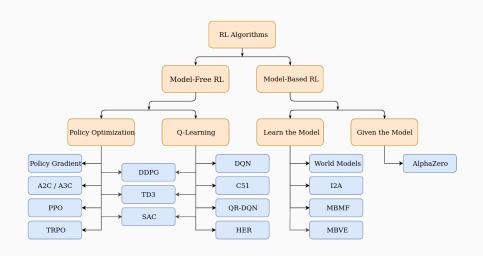
A reinforcement-learning algorithm may include any of the following:

- · Policy: function approximator for the agent's behavior
- · Value function: function approximator for the value of states
- Model: representation of the environment

An algorithm with a policy (actor) and a value function (critic) is called *actor-critic*.

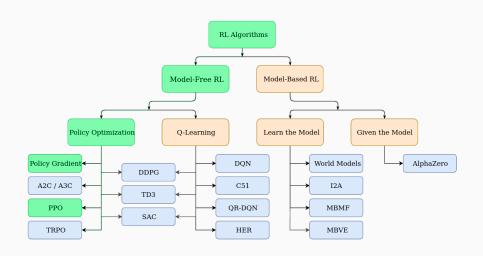
An algorithm with/without an explicit model is called *model-based/free*.

A taxonomy of RL algorithms



There are several taxonomies, none of them fully works. This one is from [Ach18].

A taxonomy of RL algorithms



Our focus in what follows.

Policy optimization

Policy-based algorithms

Policy-based algorithms update policy parameters θ iteratively. At each iteration k:

- · Collect episodes $\mathcal{D}_k = \{\tau\}$
- Update the policy $\pi_{\theta_{k+1}} = update(\pi_{\theta_k}, \mathcal{D}_k)$

Two ways to collect episodes:

- On-policy: episodes \mathcal{D}_k are collected with the latest policy π_k
- Off-policy: episodes \mathcal{D}_k are collected with any policy

Policy optimization

The goal of RL is to find a policy that maximizes the expected return. In terms of θ :

$$J(\theta) = \mathbb{E}_{\tau \sim \pi_{\theta}}[R(\tau)]$$

In policy optimization, we seek an optimum by gradient ascent:

$$\theta_{k+1} = \theta_k + \alpha \nabla_{\theta} J(\theta_k)$$

The gradient $\nabla_{\theta}J$ with respect to policy parameters θ is called the *policy gradient*.

Policy gradient theorem

Policy gradient theorem

The policy gradient can be computed from returns and the log-policy gradient $\nabla_{\theta} \log \pi_{\theta}$ as:

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\tau \sim \pi_{\theta}} \left(R(\tau) \sum_{s_t, a_t \in \tau} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \right)$$

LHS: the graal. RHS: things we observe $(R(\tau))$ or know by design $(\nabla_{\theta} \log \pi_{\theta})$.

Policy gradient theorem: proof sketch

$$\begin{split} \nabla_{\theta} J(\theta) &= \nabla_{\theta} \mathbb{E}_{\tau \sim \pi_{\theta}}(R(\tau)) & \text{definition} \\ &= \nabla_{\theta} \int_{\tau} R(\tau) \mathbb{P}(\tau|\theta) \mathrm{d}\tau & \text{expectation as integral} \\ &= \int_{\tau} R(\tau) \nabla_{\theta} \mathbb{P}(\tau|\theta) \mathrm{d}\tau & \text{Leibniz integral rule} \\ &= \int_{\tau} R(\tau) \mathbb{P}(\tau|\theta) \nabla_{\theta} \log \mathbb{P}(\tau|\theta) \mathrm{d}\tau & \text{log-derivative trick} \\ &= \int_{\tau} R(\tau) \sum_{s_{t}, a_{t} \in \tau} \nabla_{\theta} \log \pi_{\theta}(a_{t}|s_{t}) \mathbb{P}(\tau|\theta) \mathrm{d}\tau & \text{expand } \mathbb{P}(\tau|\theta) \text{ as product} \\ &= \mathbb{E}_{\tau \sim \pi_{\theta}} \left(R(\tau) \sum_{s_{t}, a_{t} \in \tau} \nabla_{\theta} \log \pi_{\theta}(a_{t}|s_{t}) \right) & \text{integral as expectation} \end{split}$$

REINFORCE (1/2)

REINFORCE algorithm [SB18]

```
\begin{array}{l} \text{Data: initial policy parameters $\theta_0$, learning rate $\alpha$} \\ \text{Initialize policy parameters $\theta$ (e.g. to 0);} \\ \text{for $k=0,1,2,\dots$ do} \\ \text{Roll out an episode $\tau=(o_0,a_0,\dots,o_N,a_N)$ following $\pi_{\theta_k}$;} \\ \text{for each step $t\in\tau$ do} \\ \text{$\left|\begin{array}{c} R\leftarrow\sum_{t'=t+1}^{N}\gamma^{t'-t-1}r_{t'};\\ \theta\leftarrow\theta+\alpha\gamma^tR\nabla_\theta\log\pi_\theta(a_t|s_t) \end{array}\right.} \\ \text{end} \\ \text{end} \end{array}
```

REINFORCE (2/2)

Gradient ascent:

$$\theta_{k+1} = \theta_k + \alpha \nabla_{\theta} J(\theta_k)$$

From the policy gradient theorem, this is equivalent to:

$$\theta_{k+1} = \theta_k + \alpha \mathbb{E}_{\tau \sim \pi_{\theta}} \left(R(\tau) \sum_{s_t, a_t \in \tau} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \right)$$

REINFORCE drops the expectation:

$$\theta_{k+1} = \theta_k + \alpha R(\tau_k) \sum_{s_t, a_t \in \tau_k} \nabla_{\theta} \log \pi_{\theta}(a_t|s_t)$$

Vanilla policy gradient [Ach18]

Data: initial policy parameters θ_0 , initial value function parameters ϕ_0 , learning rate α for $k=0,1,2,\ldots$ do

Collect episodes $\mathcal{D}_k = \{\tau_i\}$ by running $\pi_{\theta} = \pi(\theta_k)$;

Compute returns \hat{R}_t and advantage estimates \hat{A}_t based on V_{ϕ_k} ;

Estimate the policy gradient as

$$\hat{g}_k = \frac{1}{|\mathcal{D}_k|} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t|s_t)|_{\theta_k} \hat{A}_t$$

Update policy parameters by e.g. gradient ascent, $\theta_{k+1} = \theta_k + \alpha \hat{g}_k$; Fit value function by regression on mean-square error:

$$\phi_{k+1} = \arg\min_{\phi} \frac{1}{T|\mathcal{D}_k|} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \left(\hat{R}_t - V_{\phi}(s_t)\right)^2$$

end

Proximal policy optimization [Sch+17]

Data: initial policy parameters $heta_0$, initial value function parameters ϕ_0

for
$$k = 0, 1, 2, ...$$
 do

Collect episodes $\mathcal{D}_k = \{\tau_i\}$ by running $\pi_{\theta} = \pi(\theta_k)$;

Compute returns \hat{R}_t and advantage estimates \hat{A}_t based on V_{ϕ_k} ;

Clipping: Update policy parameters by maximizing the clipping objective:

$$\theta_{k+1} = \arg\max_{\theta} \frac{1}{|\mathcal{D}_k|T} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \min\left(\frac{\pi_{\theta}(a_t|s_t)}{\pi_{\theta_k}(a_t|s_t)} A^{\pi_{\theta_k}}(s_t, a_t), g(\epsilon, A^{\pi_{\theta_k}}(s_t, a_t))\right)$$

where $g(\epsilon, A) = (1 + \epsilon)A$ if $A \ge 0$ else $(1 - \epsilon)A$

Fit value function by regression on mean-square error:

$$\phi_{k+1} = \arg\min_{\phi} \frac{1}{T|\mathcal{D}_k|} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \left(\hat{R}_t - V_{\phi}(s_t)\right)^2$$

end

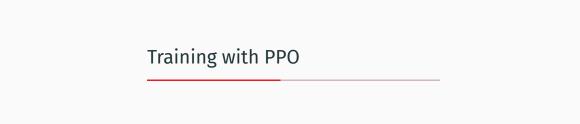
Intuition behind clipping

When the advantage is positive:

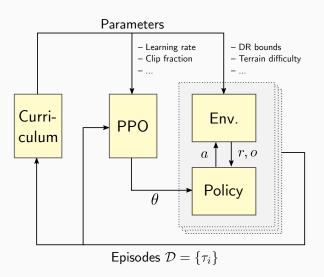
$$L(s, a, \theta_k, \theta) = \min\left(\frac{\pi_{\theta}(a|s)}{\pi_{\theta_k}(a|s)}, (1+\epsilon)\right) A^{\pi_{\theta_k}}(s, a)$$

The objective increases if the action becomes more likely $\pi_{\theta}(a|s) > \pi_{\theta_k}(a|s)$, but no extra benefit as soon as $\pi_{\theta}(a|s) > (1 + \epsilon)\pi_{\theta_k}(a|s)$.

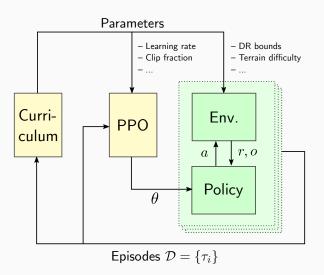
When the advantage is negative: idem mutatis mutandis.



Training



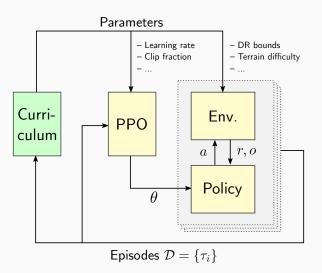
Environment



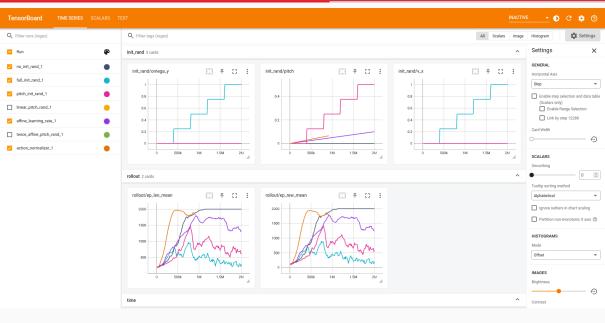
Rolling out episodes with a simulator

```
nd ./tools/bazel run //agents/ppo balancer:train -- --nb-envs 6 --show
     Analyzed target //agents/ppo balancer:train (108 packages loaded, 17832 targets configured).
2023-11-14 11:31:04.519] [info] To track in TensorBoard, run `tensorboard --logdir /home/scaron/src/upkie/training/2023-11-14` (train.pv:366)
2023-11-14 11:31:04,524] [info] New policy name is "marshiest" (train.py:236)
[2023-11-14 11:31:04.550] [info] Waiting for spine /monogamous to start (trial 1 / 10)... (spine interface.py:46)
2023-11-14 11:31:04.554] [info] Command line: shm name = /monogamous
2023-11-14 11:31:04.554] [info] Command line: nb substeps = 5
2023-11-14 11:31:04.554] [info] Command line: spine_frequency = 1000 Hz
[2023-11-14 11:31:04.554] [warning] [Joystick] Observer disabled: no loystick found at /dev/input/is0
started thread 0
```

Curriculum



Monitoring training



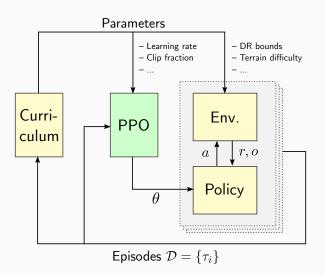
Episode metrics

Two main episode metrics:

- ep_len_mean: average length of an episode, in number of environment steps.
- ep_rew_mean: average return of an episode.

If training goes well, both eventually plateau at their maximum values.

Training with PPO



Monitoring PPO



PPO loss function

Surrogate loss of PPO

```
loss = policy_gradient_loss + ent_coef * entropy_loss + vf_coef * value_loss
```

- policy_gradient_loss: regular loss resulting from episode returns.
- entropy_loss: negative of the average policy entropy. It should increase to zero over training as the policy becomes more deterministic.
- value_loss: value function estimation loss, i.e. error between the output of the function estimator and Monte-Carlo or TD(GAE lambda) estimates.

PPO hyperparameters

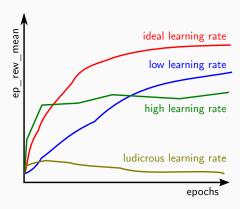
The PPO implementation in Stable Baselines3 has > 25 parameters, including:

- clip_range: clipping factor in policy loss.
- ent_coef: weight of entropy term in the surrogate loss.
- gae_lambda: parameter of Generalized Advantage Estimation.
- net_arch_pi: policy network architecture.
- net_arch_vf: value network architecture.
- normalize_advantage: use advantage normalization?
- vf_coef: weight of value-function term in the surrogate loss.

Optimizer parameters: steps, epochs, mini-batching

The optimizer behind PPO, usually Adam [KB14], comes with parameters:

- · learning_rate : step size parameter, typically decreasing with a linear schedule.
- n_steps : number of environment steps to collect per rollout buffer.
- n_epochs : number of uses of the rollout buffer while optimizing the surrogate loss.
- batch_size : mini-batch size, same as in Stochastic Gradient Descent.



PPO health metrics

Finally, some metrics indicate whether training is going well:

- approx_kl : approximate KL divergence between the old policy and the new one.
- clip_fraction: mean fraction of policy ratios that were clipped.
- clip_range: value of the clipping factor for the policy ratios.
- explained_variance : pprox 1 when the value function is a good predictor for returns.

Application to robotics

Training a policy

General things to do when training a policy:

- Augment observations with history
- · Observation-action normalization
- Curriculum learning

Augmenting observations with history

(Welcome to the worst slide of this deck!)

- · Lag of a system: number of observations required to estimate its state
- \cdot We assume the Markov property: lag = 1
- · Delays in control and physical systems increase their lag
- · Counter-measure: augment observations with history, restore the Markov property

RL-trained policies can be remarkable state estimators!

Observation-action normalization

Specific to deep RL, normalize the:

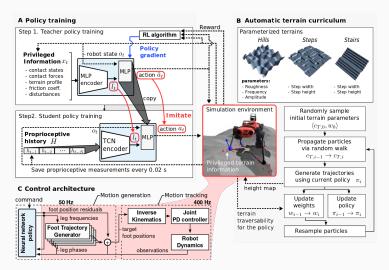
- · Observation space: bound physical quantities, when possible.
- Action space: rescaling to [-1,1] is common practice.

Unnormalized actions don't work well on actors with Gaussian outputs (as in PPO):

- Limits too large \Rightarrow sampled actions cluster around zero.
- Limits too small \Rightarrow sampled actions saturate all the time, bang-bang behavior.

Curriculum learning

Domain randomization and task difficulty vary based on policy performance. Example: terrain curriculum for quadrupedal locomotion [Lee+20]:



Sim-to-real gap

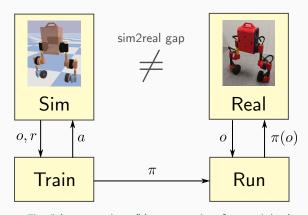


Figure 1: The "sim-to-real gap" is a metaphor for model mismatch.

Crossing the sim-to-real gap

To improve state observation:

· Teacher-student distillation

To learn a more robust² policy:

- · Domain randomization
- · Data-based simulation
- · Reward shaping

²Robustness over model variations.

Domain randomization

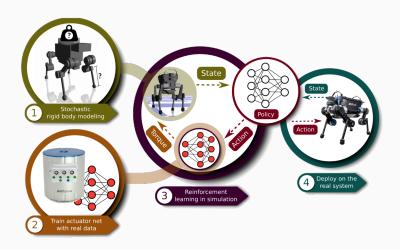
Randomize selected environment parameters:

- · Robot geometry: limb lengths, wheel diameters, ...
- · Inertias: masses, mass distributions
- Initial state: $s_0 \sim \rho_0(\cdot)$
- · Actuation models: delays, bandwidth, ...
- **Perturbations:** send $(1 \pm \epsilon)\tau$ torques...

Domain randomization makes policies more conservative.

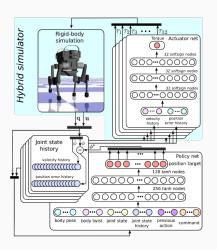
It slows down or completely hampers training.

Data-based actuation models (1/2)



³Jemin Hwangbo, Joonho Lee, Alexey Dosovitskiy, Dario Bellicoso, Vassilios Tsounis, Vladlen Koltun, and Marco Hutter. "Learning agile and dynamic motor skills for legged robots". In: *Science Robotics* 4.26 (2019).

Data-based actuation models (2/2)



⁴Jemin Hwangbo, Joonho Lee, Alexey Dosovitskiy, Dario Bellicoso, Vassilios Tsounis, Vladlen Koltun, and Marco Hutter. "Learning agile and dynamic motor skills for legged robots". In: *Science Robotics* 4.26 (2019).

Reward shaping

Let r_e denote the reward associated with an error function e:

Motivation:

• Exponential: $r_e = \exp(-e^2)$

Penalization:

- Absolute value $r_e = -|e|$
- Squared value: $r_e = -e^2$

RewArt

Making an RL pipeline work can lead to complex rewards, e.g. in [Lee+20]:

- Linear velocity tracking: $r_{lv} = \exp(-2.0(v_{pr}-0.6)^2)$, or 1, or 0
- Angular velocity tracking: $r_{av} = \exp(-1.5(\omega_{pr} 0.6)^2)$, or 1
- Base motion tracking: $r_b = \exp(-1.5v_o^2) + \exp(-1.5\|(^B_{IB}\omega)_{xy}\|^2)$
- Foot clearance: $r_{fc} = \sum_{i \in I_{swing}} \mathbf{1}_{fclear}(i) / |I_{swing}|$
- Body-terrain collisions: $r_{bc} = -|I_{c,body} \setminus I_{c,foot}|$
- Foot acceleration smoothness: $r_s = -\|(r_{f,d})_t 2(r_{f,d})_{t-1} + (r_{f,d})_{t-2}\|$
- \cdot Torque penalty: $r_{ au} = -\sum_{i} | au_{i}|$

Final reward: $r = 0.05r_{lv} + 0.05r_{av} + 0.04r_b + 0.01r_{fc} + 0.02r_{bc} + 0.025r_s + 2 \cdot 10^{-5}r_{\tau}$

Example: Upkie ground velocity

Observation:

Index	Symbol	Description
0	θ	pitch from torso to inertial frame
1	p	ground position
2	$\dot{ heta}$	pitch angular velocity from torso to inertial frame
3	\dot{p}	ground velocity

Action:

Index	Symbol	Description
0	v	commanded ground velocity

Example: Rewards

RewArt is not a necessity: advocacy for simple rewards.

In the Upkie ground-velocity environment, we went for:

$$p_{tip} = p + \ell \sin(\theta)$$
$$r(o) := \exp\left(-\left(\frac{p_{tip}}{\sigma}\right)^2\right)$$

Including higher-order derivatives does not always help.

Example: environment wrappers

Composability of environments helps while prototyping. In the Upkie ground-velocity environment:

- · Base environment: velocity commands
- · (Training:) add action noise
- · (Training:) add observation noise
- · (Training:) low-pass filter on wheel actuators
- · Include last ten observation-action pairs in observation vector
- Change action to ground acceleration (with bounds)
- \cdot Rescale action between -1 and 1

Keep in mind that we are in a stochastic world

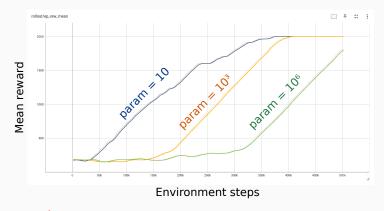


Figure 2: We may be observing the effect of our parameter.

Keep in mind that we are in a stochastic world



Figure 2: Or we may be observing the variance of the training process.

Challenges for RL

- · Sim-to-real: no one-size-fits-all method, depends on robot and task
- · Jittering: action noise, due to need for stochastic policies (PGT!)
- Sample efficiency: requires millions of samples (s_t, a_t, s_{t+1})
- · Reward shaping: often helps, not a necessity, "RewArt"!
- Curriculum learning: humans monitoring computers that learn⁵

⁵Also known as SGD: "Student Grad Descent"

What did we see?

What we saw

Introduction to policy optimization:

- Partially-observable Markov decision process (POMDP)
- · The goal of reinforcement learning
- · Model, policy and value function
- · Policy optimization: REINFORCE, policy gradient, PPO

Application to robotics:

- · Training policies: add history, normalization, curriculum
- · Sim-to-real gap: domain randomization, data-based sim, "RewArt"

RL is not magic: great results, possibly going to great lengths!

Thank you for your attention!⁶

Thanks to Elliot Chane-Sane, Thomas Flayols, Nicolas Perrin-Gilbert, Philippe Souères and the 2023 class at MVA for feedback on previous versions of these slides.



References i

[Ach18]

	https://spinningup.openai.com/.2018.
[ACN10]	Pieter Abbeel, Adam Coates, and Andrew Y Ng. "Autonomous helicopter aerobatics through apprenticeship learning". In: <i>The International Journal of Robotics Research</i> 29.13 (2010), pp. 1608–1639.
[And+20]	OpenAl: Marcin Andrychowicz, Bowen Baker, Maciek Chociej, Rafal Jozefowicz, Bob McGrew, Jakub Pachocki, Arthur Petron, Matthias Plappert, Glenn Powell, Alex Ray, et al. "Learning dexterous in-hand manipulation". In: <i>The International Journal of Robotics Research</i> 39.1 (2020), pp. 3–20.
[AS97]	Christopher G Atkeson and Stefan Schaal. "Robot learning from demonstration". In: <i>ICML</i> . Vol. 97. 1997, pp. 12–20.
[Hwa+19]	Jemin Hwangbo, Joonho Lee, Alexey Dosovitskiy, Dario Bellicoso, Vassilios Tsounis, Vladlen Koltun, and Marco Hutter. "Learning agile and dynamic motor skills for legged robots". In: <i>Science Robotics</i> 4.26 (2019).
[KB14]	Diederik P Kingma and Jimmy Ba. "Adam: A method for stochastic optimization". In: arXiv preprint arXiv:1412.6980 (2014).
[Lee+20]	Joonho Lee, Jemin Hwangbo, Lorenz Wellhausen, Vladlen Koltun, and Marco Hutter. "Learning quadrupedal locomotion over challenging terrain". In: <i>Science robotics</i> 5.47 (2020).

Josh Achiam. Spinning Up in Deep Reinforcement Learning.

References ii

[SB18]	Richard S Sutton and Andrew G Barto. Reinforcement learning: An introduction. MIT press, 2018.
[Sch+17]	John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. "Proximal policy optimization algorithms". In: <i>arXiv preprint arXiv:1707.06347</i> (2017).