3D Bipedal Walking including COM height variations

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What do we want?

COMANOID project – `https://comanoid.cnrs.fr`

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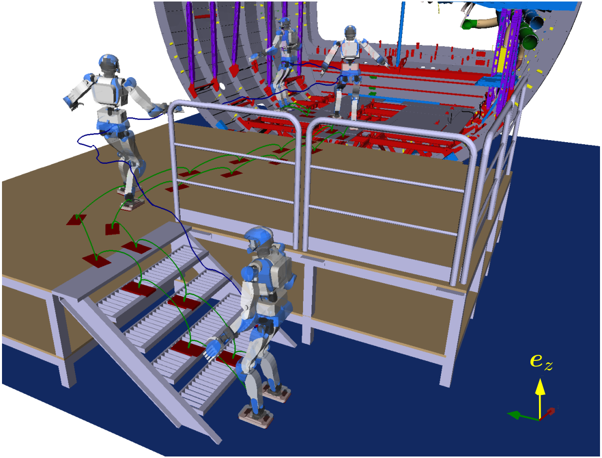
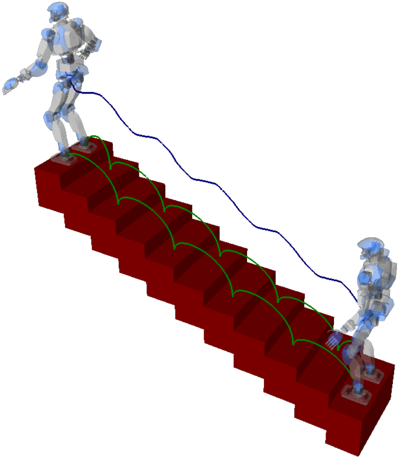


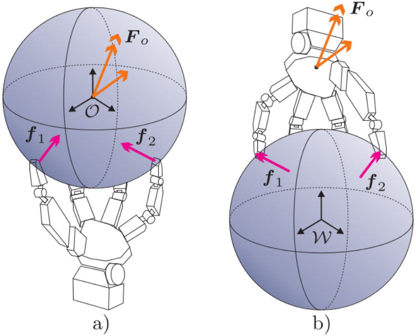
COMANOID project – Aircraft entry plan (2017)

What do we want?



Hard part: dynamic stair climbing

Interest reaches farther than humanoids



Duality between manipulation and walking

Figures adapted from [Eng+11] (left) and [HRO16] (right)

How to walk a humanoid robot?

- Walking pattern generation (= planning)
- Walking stabilization (= tracking)

How to walk a humanoid robot?

- **Walking pattern generation (= planning)**
- Walking stabilization (= tracking)

Walking on a plane

- Newton equation:

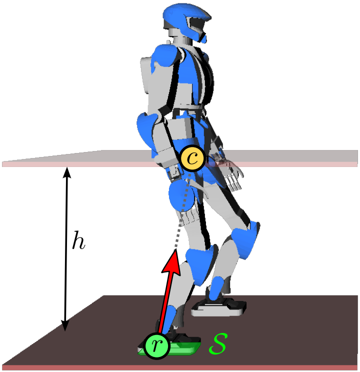
$$m\ddot{c} = f + m\vec{g}$$

- Force parameterization:

$$f = \omega^2(c - r)$$

- c: center of mass
- r: center of pressure
- In the horizontal plane:

$$\ddot{c}^{xy} = \omega^2(c^{xy} - r^{xy})$$



Walking on a plane

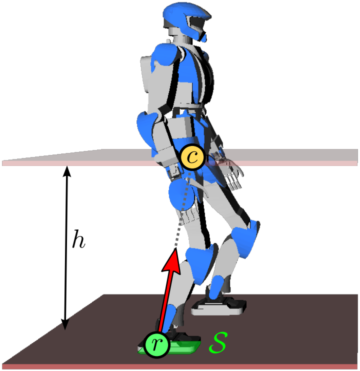
- Linear time-invariant system:

$$\ddot{c} = \omega^2(c - r)$$

- Plus, feasibility constraint:

$$r \in \mathcal{S}$$

- **Question:** how to stop?



Walking on a plane

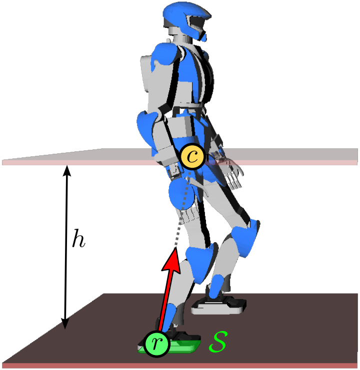
- **System:** $x = \begin{bmatrix} c & \dot{c} \end{bmatrix}$ where

$$\ddot{c} = \omega^2 (c - r)$$

- **Input:** center of pressure r
- **Balance:** starting from

$$x_0 = \begin{bmatrix} c_0 \\ \dot{c}_0 \end{bmatrix}$$

How to bring the system to a stop with a stationary input?



Divergent Component of Motion

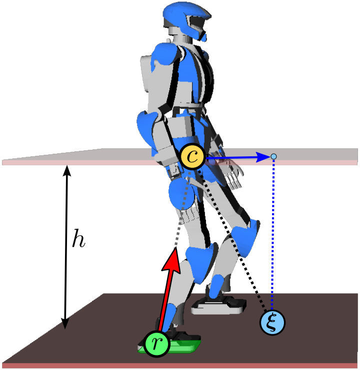
- Recall that $\ddot{c} = \omega^2(c - r)$
- Divergent comp. of motion:

$$\xi = c + \frac{\dot{c}}{\omega}$$

- First-order dynamics:

$$\dot{\xi} = \omega(\xi - r)$$
$$\dot{c} = \omega(\xi - c)$$

- Stopped by $r = \xi$ (stationary)



On this topic, go and read [Eng+11]

Trajectory, or not trajectory?

Trajectory-based:

Trajectory-free:

Best reference here: [Wie06]

Trajectory, or not trajectory?

Trajectory-based:

- Interpolate *e.g.*

$$c(t) \;=\; \sum_i a_i t^i$$

- Boundary conditions
- Center of pressure is:

$$r(t) = c(t) - \ddot{c}(t)/\omega^2$$

- Feasibility: $Ar(t) \leq b$?
- Enforcing boundary conditions is easy, but enforcing dynamic feasibility is not

Trajectory-free:

Best reference here: [Wie06]

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Best reference here: [Wie06]

Trajectory-free:

- Discretize: for $t \in [t_k, t_{k+1})$,

$$\ddot{c}(t) = \ddot{c}_k$$

- Boundary conditions
- Center of pressure is:

$$r_k = c_k - \ddot{c}_k/\omega^2$$

- Feasibility: $Ar_k \leq b$
- Solve a quadratic program:

$$\begin{aligned} \min_{\{\ddot{c}_k\}}. \quad & \sum_k \|\ddot{c}_k\|^2 \\ \text{s.t.} \quad & \text{above constraints} \end{aligned}$$

Roadmap

Stair climbing:

- Planar walking $\rightarrow$ variable height
- Look into the future $\rightarrow$ divergent component of motion?
- Minimize height variations $\rightarrow$ trajectory free

Variable-height extension

- Newton equation:
$$\ddot{c} = \lambda(c - r) + g$$
- Divergent component of motion:
$$\xi = \dot{c} + \omega c$$
- First-order dynamics:
$$\dot{\xi} = \omega\xi + g - \lambda r$$
- ... under the Riccati equation: $\dot{\omega} = \omega^2 - \lambda$

Discussed in [CM18; Car+18]

Boundedness condition

- Differential equation: $\dot{\xi} = \omega\xi + g - \lambda r$

Boundedness condition

- Differential equation: $\dot{\xi} = \omega\xi + g - \lambda r$
- Solution is:

$$\xi(t) = e^{\Omega(t)} \left(\xi(0) + \int_0^t e^{-\Omega(\tau)} (\lambda(\tau) r(\tau) - g) \mathrm{d}\tau \right)$$

- As $t \to \infty$, the divergent component ξ should stay *bounded*
- *Therefore*,

$$\xi(0) = \int_0^\infty (\lambda(t) r(t) - g) e^{-\Omega(t)} \mathrm{d}t$$

- Constraint between current state (LHS) and *all* future inputs $\lambda(t), r(t)$ of the inverted pendulum (RHS)

Optimization problem

$$\underset{\varphi_1,\dots,\varphi_N}{\text{minimize}} \quad \boxed{\sum_{j=1}^{N-1}\left[\frac{\varphi_{j+1}-\varphi_j}{\Delta_j}-\frac{\varphi_j-\varphi_{j-1}}{\Delta_{j-1}}\right]^2} \tag{1}$$

$$\text{subject to} \quad \boxed{\sum_{j=0}^{N-1}\frac{\Delta_j}{\sqrt{\varphi_{j+1}}+\sqrt{\varphi_j}}-\frac{c_i^z}{g}\sqrt{\varphi_N}=\frac{\dot{c}_i^z}{g}} \tag{2}$$

$$\boxed{\omega_{i,\min}^2 \leq \varphi_N \leq \omega_{i,\max}^2} \tag{3}$$

$$\boxed{\forall j,\ \lambda_{\min}\Delta_j \leq \varphi_{j+1}-\varphi_j \leq \lambda_{\max}\Delta_j} \tag{4}$$

$$\boxed{\varphi_1 = \Delta_0 g / z_{\mathrm{f}}} \tag{5}$$

(1): min. height variations (2): boundedness (3): CoP polygon
(4): pressure constraints (5): stationary height z_{f}

Behavior of solutions

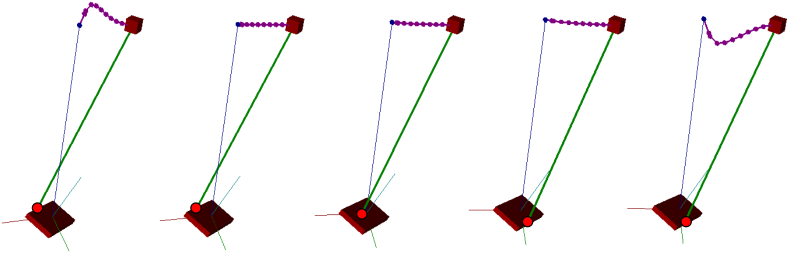
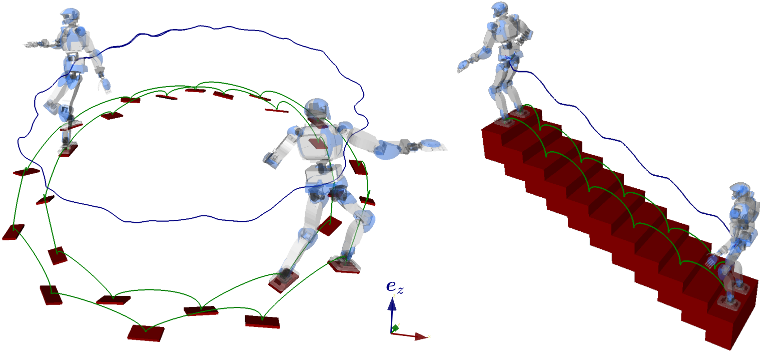


Figure : CoM trajectories obtained by solving the resultant nonlinear optimization for different initial velocities.

Resulting walking patterns



Code: `https://github.com/stephane-caron/capture-walking`

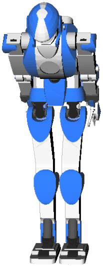
What did we see?

- Horizontal walking $\rightarrow$ force parameterization
- Dynamic motions $\rightarrow$ present/future system dynamics
- Trajectory-free optimization $\rightarrow$ find solutions from constraints
- *Outcome:* dynamic stair-climbing walking patterns

For details, see [CM18; Car+18]

Thanks!

Thank you for your attention!



References I

[Car+18] Stéphane Caron, Adrien Escande, Leonardo Lanari, and Bastien Mallein. "Capturability-based Analysis, Optimization and Control of 3D Bipedal Walking". In: Submitted. 2018. URL: https://hal.archives-ouvertes.fr/hal-01689331/document.

[CM18] Stéphane Caron and Bastien Mallein. "Balance control using both ZMP and COM height variations: A convex boundedness approach". to be presented at ICRA 2018. May 2018. URL: https://hal.archives-ouvertes.fr/hal-01590509/document.

[Eng+11] Johannes Englsberger, Christian Ott, Maximo Roa, Alin Albu-Schäffer, Gerhard Hirzinger, et al. "Bipedal walking control based on capture point dynamics". In: *Intelligent Robots and Systems (IROS), 2011 IEEE/RSJ International Conference on*. IEEE, 2011, pp. 4420–4427.

References II

[HRO16] Bernd Henze, Máximo A. Roa, and Christian Ott. "Passivity-based whole-body balancing for torque-controlled humanoid robots in multi-contact scenarios". In: *The International Journal of Robotics Research* (July 12, 2016). DOI: 10.1177/0278364916653815.

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