3D Bipedal Walking including COM height variations

Stéphane Caron

CRI Group Seminar Series

May 14, 2018

What do we want?

COMANOID project – `https://comanoid.cnrs.fr`

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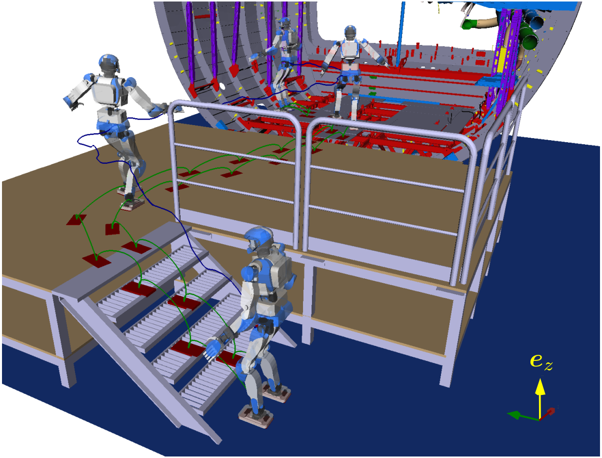

COMANOID project – Aircraft entry plan (2017)

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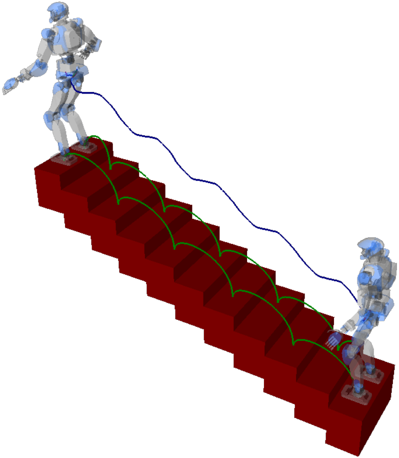
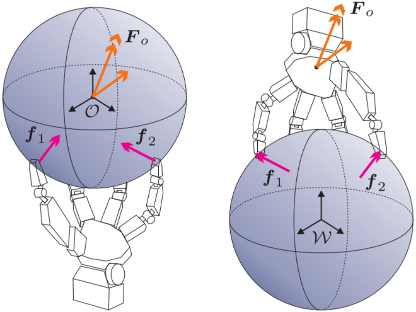


Hard part: dynamic stair climbing

Interest reaches farther than humanoids



a) b)

Duality between manipulation and walking

Figures adapted from [Eng+11] (left) and [HRO16] (right)

How to walk a humanoid robot?

- Walking pattern generation (= planning)
- Walking stabilization (= tracking)

How to walk a humanoid robot?

- **Walking pattern generation (= planning)**
- Walking stabilization (= tracking)

Walking on a plane

- Newton equation:

$$m\ddot{c} = f + m\vec{g}$$

- Force is grounded at CoP:

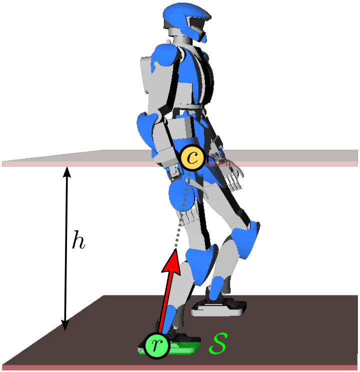
$$f = \omega^2(c - r)$$

- Holonomic constraint:

$$\ddot{c}^z = 0 \;\Rightarrow\; \omega^2 = g/h$$

- Newton equ. simplifies to:

$$\ddot{c}^{xy} = \omega^2(c^{xy} - r^{xy})$$



Walking on a plane

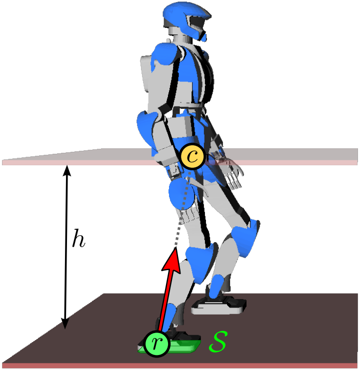
- Linear time-invariant system:

$$\ddot{c} = \omega^2 (c - r)$$

- Plus, feasibility constraint:

$$r \in \mathcal{S}$$

- **Question:** how to stop?



Walking on a plane

- **System:** $x = \begin{bmatrix} c & \dot{c} \end{bmatrix}$ where

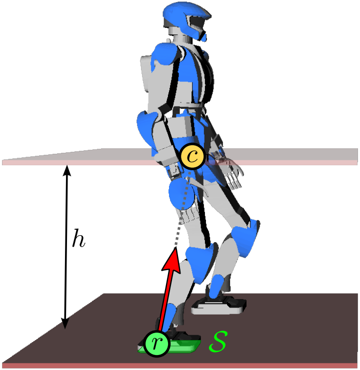
$$\ddot{c} = \omega^2 (c - r)$$

- **Input:** center of pressure r
- **Balance:** starting from

$$x_0 = \begin{bmatrix} c_0 \\ \dot{c}_0 \end{bmatrix}$$

How to bring the sys. to a stop?

With a stationary solution?



Instantaneous Capture Point

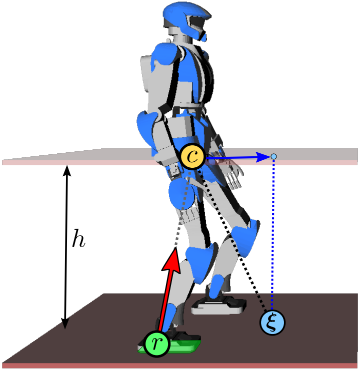
- Recall that $\ddot{c} = \omega^2(c - r)$
- Define the *capture point*:

$$\xi = c + \frac{\dot{c}}{\omega}$$

- First-order dynamics:

$$\dot{\xi} = \omega(\xi - r)$$
$$\dot{c} = \omega(\xi - c)$$

- Stopped by $r = \xi$ (stationary)



On this topic, go and read [Eng+11]

Towards 3D, take one

- Apply same equation but in 3D:

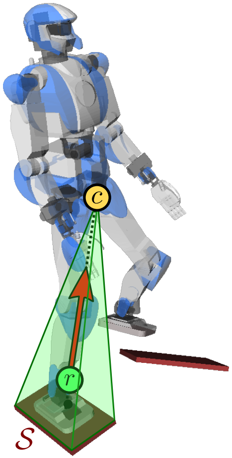
$$\ddot{c} = \omega^2(c - \nu)$$

ν: Virtual Repellent Point

- Feasibility constraint becomes:

$$r = \nu + \frac{g}{\omega^2} \in \mathcal{S}$$

- Equation of motion is LTI but system nonlinear from feasibility constraint



Related references: [EOA15; CK17]

Time-varying DCM

- Newton equation:

$$\ddot{c} = \lambda(c - r) + g$$

- Divergent component of motion:

$$\xi = \dot{c} + \omega c$$

- First-order dynamics:

$$\dot{\xi} = \omega \xi + g - \lambda r$$

- ... under the Riccati equation: $\dot{\omega} = \omega^2 - \lambda$

Discussed in [CM18; Car+18]

Boundedness condition

- Differential equation: $\dot{\xi} = \omega\xi + g - \lambda r$

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- As $t \to \infty$, the DCM ξ should stay *bounded*
- *Therefore,*

$$\xi(0) = \int_0^\infty (\lambda(t) r(t) - g) e^{-\Omega(t)} \mathrm{d}t$$

- Constraint between current state (LHS) and *all* future inputs $\lambda(t), r(t)$ of the inverted pendulum (RHS)

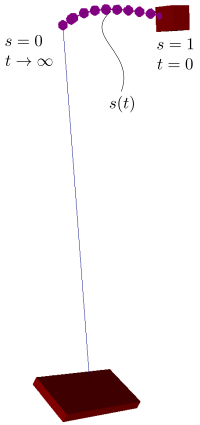
Problem formulation

- Change of variable: $s(t) = e^{-\Omega(t)}$
- Boundedness condition becomes:

$$\int_0^1 r^{xy}(s)(s\omega(s))' \mathrm{d}s = \dot{c}_i^{xy} + \omega_i c_i^{xy}$$

$$g \int_0^1 \frac{1}{\omega(s)} \mathrm{d}s = \dot{c}_i^{z} + \omega_i c_i^{z}$$

- Optimize over $\varphi_i = s_i^2 \omega(s_i)^2$
- From φ^*, derive $\lambda(s), \omega(s), \lambda(t), \omega(t), r(t), c(t), \ldots$



Optimization problem

$$\underset{\varphi_1,\dots,\varphi_N}{\text{minimize}} \quad \boxed{\sum_{j=1}^{N-1} \left[\frac{\varphi_{j+1} - \varphi_j}{\Delta_j} - \frac{\varphi_j - \varphi_{j-1}}{\Delta_{j-1}} \right]^2} \tag{1}$$

$$\text{subject to} \quad \boxed{\sum_{j=0}^{N-1} \frac{\Delta_j}{\sqrt{\varphi_{j+1}} + \sqrt{\varphi_j}} - \frac{c_i^z}{g}\sqrt{\varphi_N} = \frac{\dot{c}_i^z}{g}} \tag{2}$$

$$\boxed{\omega_{i,\min}^2 \leq \varphi_N \leq \omega_{i,\max}^2} \tag{3}$$

$$\boxed{\forall j, \ \lambda_{\min}\Delta_j \leq \varphi_{j+1} - \varphi_j \leq \lambda_{\max}\Delta_j} \tag{4}$$

$$\boxed{\varphi_1 = \Delta_0 g / z_f} \tag{5}$$

(1): min. height variations (2): boundedness (3): CoP polygon
(4): pressure constraints (5): stationary height z_f

Behavior of solutions

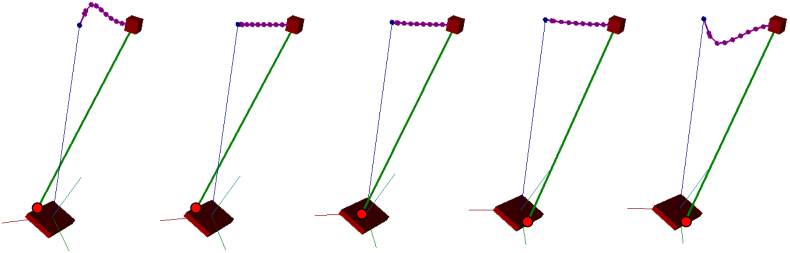
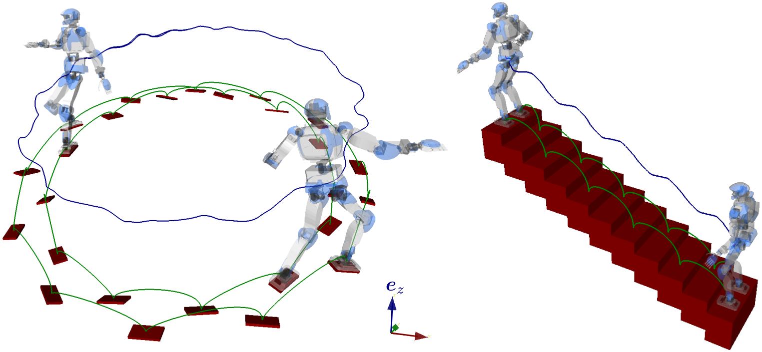


Figure : CoM trajectories obtained by solving the resultant nonlinear optimization for different initial velocities.

Resulting walking patterns



Code: `https://github.com/stephane-caron/capture-walking`

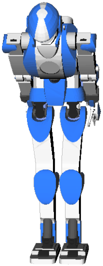
What did we see?

- Horizontal walking $\rightarrow$ LTI system
- With CoM height variations $\rightarrow$ nonlinear system
- Solve first the *boundedness condition* $\rightarrow$ LTV system
- Link with TOPP, nonlinear optimization...
- *Outcome:* dynamic stair-climbing walking patterns

For details, see [CM18; Car+18]

Thanks!

Thank you for your attention!



References I

[Car+18] Stéphane Caron, Adrien Escande, Leonardo Lanari, and Bastien Mallein. "Capturability-based Analysis, Optimization and Control of 3D Bipedal Walking". In: Submitted. 2018. URL: https://hal.archives-ouvertes.fr/hal-01689331/document.

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[CM18] Stéphane Caron and Bastien Mallein. "Balance control using both ZMP and COM height variations: A convex boundedness approach". to be presented at ICRA 2018. May 2018. URL: https://hal.archives-ouvertes.fr/hal-01590509/document.

References II

[Eng+11] Johannes Englsberger, Christian Ott, Maximo Roa, Alin Albu-Schäffer, Gerhard Hirzinger, et al. “Bipedal walking control based on capture point dynamics”. In: *Intelligent Robots and Systems (IROS), 2011 IEEE/RSJ International Conference on*. IEEE, 2011, pp. 4420–4427.

[EOA15] Johannes Englsberger, Christian Ott, and Alin Albu-Schaffer. “Three-dimensional bipedal walking control based on divergent component of motion”. In: *IEEE Transactions on Robotics* 31.2 (2015), pp. 355–368.

[HRO16] Bernd Henze, Máximo A. Roa, and Christian Ott. “Passivity-based whole-body balancing for torque-controlled humanoid robots in multi-contact scenarios”. In: *The International Journal of Robotics Research* (July 12, 2016). DOI: 10.1177/0278364916653815.