Balancing legged robots

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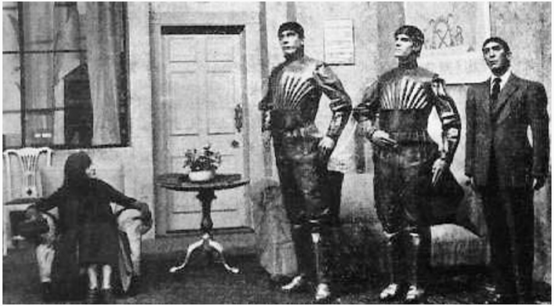
December 16, 2023

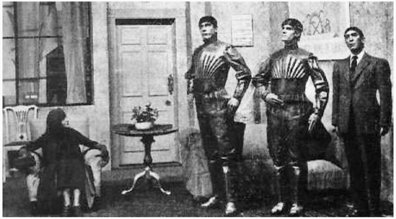
Inria, École normale supérieure

Foreword



Rossumovi Univerzální Roboti, Karel Čapek, 1921.

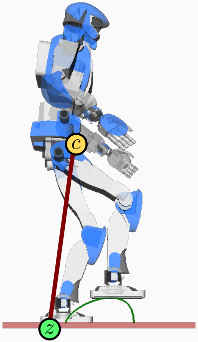
DARPA Robotics Challenge, Finals, 2015.



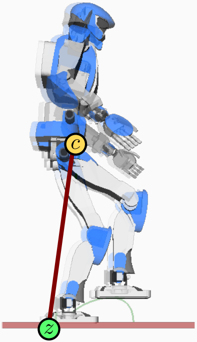
[1] Emanuel Todorov. "Goal directed dynamics". In: *2018 IEEE International Conference on Robotics and Automation*. 2018, pp. 2994–3000.

Physics of balancing

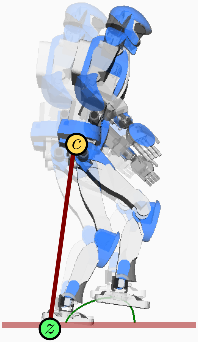
Motivation



Plan$(t + \Delta t)$



Simulation$(t + \Delta t)$

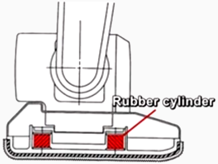


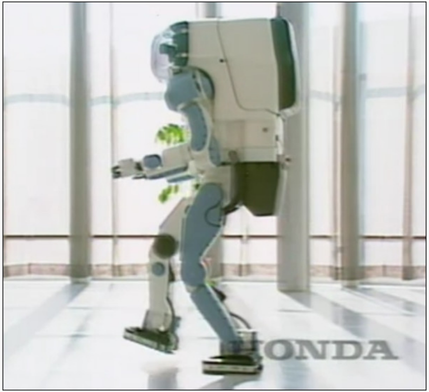
Real$(t + \Delta t)$

Honda P2

Public demonstration in 1998:

- “Zero” Moment Point (ZMP) control
- Ground reaction force control
- Impact absorption:





[2] Kazuo Hirai, Masato Hirose, Yuji Haikawa, and Toru Takenaka. “The development of Honda humanoid robot”. In: *IEEE International Conference on Robotics and Automation*. 1998.

Physical model that makes it walk

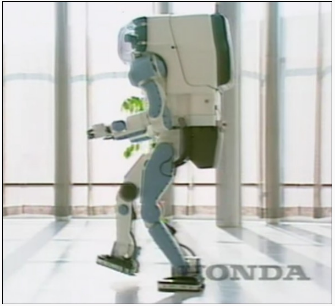


Figure 1: Honda P2 walking

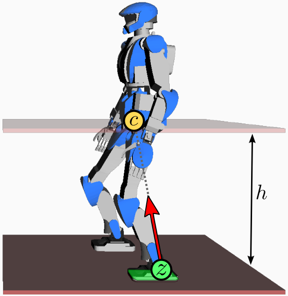


Figure 2: Model it relies on

Centroidal dynamics

Whole-body dynamics:

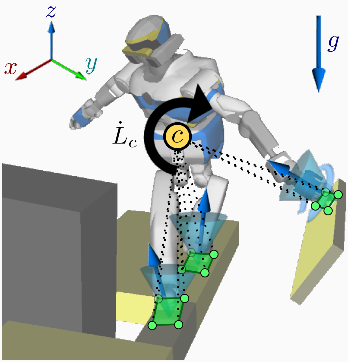
$$M\ddot{q} + N = S^T\tau + J^T f$$

Actuated joints / floating base:

$$M_a\ddot{q} + N_a = \tau + J_a^T f$$
$$M_u\ddot{q} + N_u = J_u^T f$$

Centroidal dynamics:

$$\ddot{c} = g + \frac{1}{m}\sum_{i\in\text{contacts}} f_i$$
$$\dot{L}_c = \sum_{i\in\text{contacts}} (p_i - c)\times f_i$$



[3] Hervé Audren, Joris Vaillant, Abderrahmane Kheddar, Adrien Escande, Kenji Kaneko, and Eiichi Yoshida. "Model preview control in multi-contact motion-application to a humanoid robot". In: *IEEE/RSJ International Conference on Intelligent Robots and Systems*. 2014.

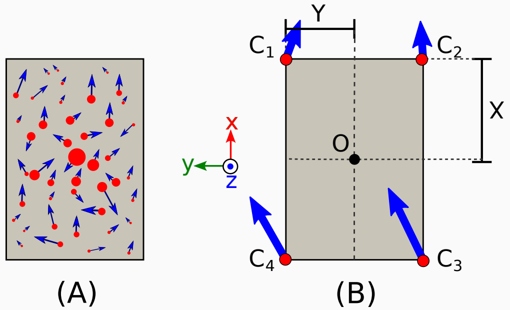


Figure 3: (A) Continuous shearing/pressure distribution under a contact surface. (B) Equivalent discrete force distribution, *i.e.* producing the same wrench.

[4] Stéphane Caron, Quang-Cuong Pham, and Yoshihiko Nakamura. "Stability of Surface Contacts for Humanoid Robots: Closed-Form Formulae of the Contact Wrench Cone for Rectangular Support Areas". In: *IEEE International Conference on Robotics and Automation*. 2015.

The **center of pressure (CoP)** is the point p on the contact surface where the resultant of *pressure* forces is applied:

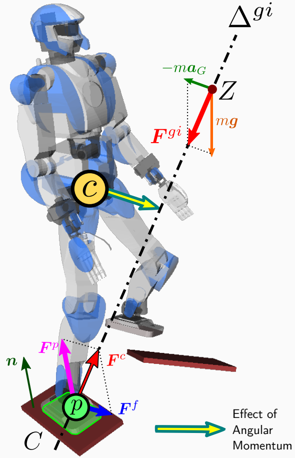
$$p = \int_{(x,y)\in\mathcal{S}} \begin{bmatrix} x \\ y \end{bmatrix} f_z(x,y)\mathrm{d}x\mathrm{d}y$$

Zero-tilting Moment Point (ZMP)

The ZMP is a point where the moment of the contact wrench is aligned with the contact normal n.

The ZMP axis intersects the contact surface at the CoP.

Informally, the net contact force is applied at the ZMP.



[5] P. Sardain and G. Bessonnet. "Forces acting on a biped robot. center of pressure-zero moment point". In: *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans* 34.5 (2004).

Reduction to a point-mass model

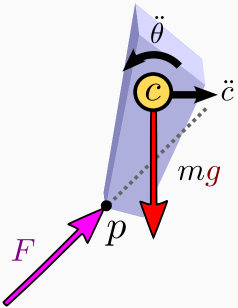


Figure 4: Net contact force does not go through CoM c $\Rightarrow \dot{L}_c = I\ddot{\theta} > 0$, body rotates and translates.

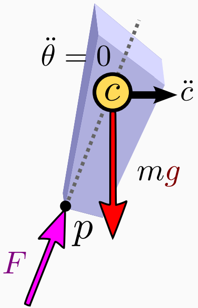


Figure 5: Net contact force goes through CoM c $\Rightarrow \dot{L}_c = 0$, body translates but does not rotate.

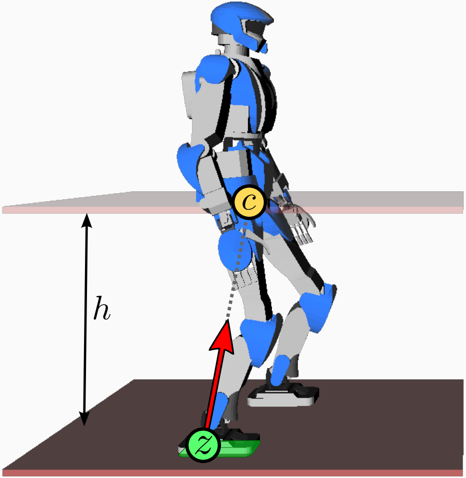
Bottom line

A constant angular momentum reduces the system to translation

Linear inverted pendulum mode

Assumptions:

- Rigid joints, sufficient power
- Conservation of angular momentum
- Constant CoM height



Equation of motion

$$\ddot{c} = \omega^2 (c - z) + g$$

- ω is a constant
- z: *zero-tilting moment point* (ZMP)
- In horiz. plane $+g$ is usually omitted

[6] Shuuji Kajita, Fumio Kanehiro, Kenji Kaneko, Kazuhito Yokoi, and Hirohisa Hirukawa. "The 3D Linear Inverted Pendulum Mode: A simple modeling for a biped walking pattern generation". In: *IEEE/RSJ International Conference on Intelligent Robots and Systems*. 2001.

Calculate the LIP constant

Equations and assumptions we have seen/made so far:

- Centroidal dynamics:

$$\ddot{c} = g + \frac{1}{m} \sum_{i \in \text{contacts}} f_i \qquad \dot{L}_c = \sum_{i \in \text{contacts}} (p_i - c) \times f_i$$

- Zero-tilting moment point:

$$\tau_z \times e_z = \left[\sum_{i \in \text{contacts}} (p_i - z) \times f_i \right] \times e_z = 0$$

- Constant height: $c \cdot e_z = h$
- Angular momentum: $\dot{L}_c = 0$

Question

Derive the LIP equation $\ddot{c} = \omega^2 (c - z) + g$. What is the expression of ω?

ZMP support area

The ZMP should lie in a support area:

$$z_{\text{min}} \leq z \leq z_{\text{max}}$$

If not, the surface contact will break into a line or a point contact.

On a flat floor and under large friction, the ZMP support area is simply the convex hull of ground contact points.[7]

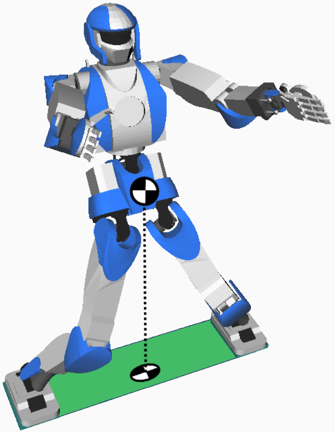


Figure 6: ZMP support area under large floor friction.

[7] This construction does not generalize to arbitrary contacts.

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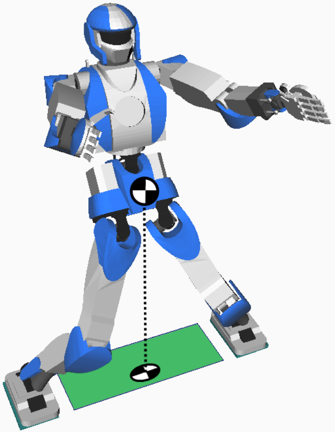


Figure 6: ZMP support area under limited floor friction.

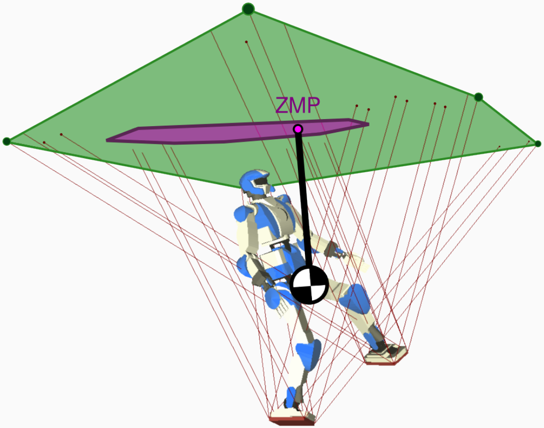
[7] This construction does not generalize to arbitrary contacts.

ZMP support area

The ZMP support area can always be computed by polyhedral geometry:

$$Gz \leq h$$

where G and h depend on the location, geometry and friction of contact surfaces.



[8] Stéphane Caron, Quang-Cuong Pham, and Yoshihiko Nakamura. “ZMP Support Areas for Multi-contact Mobility Under Frictional Constraints”. In: *IEEE Transactions on Robotics* 33.1 (2017).

Optimal control for walking

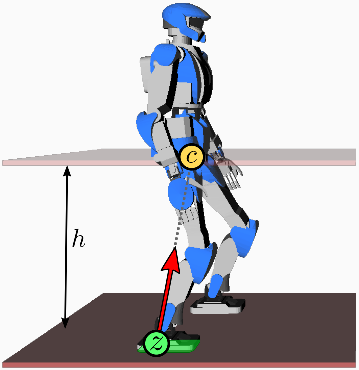
Control of a linear inverted pendulum

Continuous-time system:

- **Equation of motion:** $\ddot{c} = \omega^2 (c - z) + g$
- **Constraints:** $G z \leq h$

Problems:

- **Controlability:** find $z(t)$ such that $c(t) \to c^*$
- **Optimal control:** find $z \in \arg\min_z \int_0^T \ell(c, \dot{c}, z) \mathrm{d}t$



Linear time-invariant system

- Discretize time into steps t_k such that $t_{k+1} = t_k + T$
- Notation: $v[k] = v(t_k)$ for any quantity v
- Let's decide $\dddot{c}$ is piecewise constant:

$$
\begin{aligned}
c[k+1] &= c[k] + T\dot{c}[k] + \frac{T^2}{2}\ddot{c}[k] + \frac{T^3}{6}\dddot{c}[k] \\
\dot{c}[k+1] &= \dot{c}[k] + T\ddot{c}[k] + \frac{T^2}{2}\dddot{c}[k] \\
\ddot{c}[k+1] &= \ddot{c}[k] + T\dddot{c}[k]
\end{aligned}
$$

- Define the **state** $x[k] := \begin{bmatrix} c[k] \\ \dot{c}[k] \\ \ddot{c}[k] \end{bmatrix}$ and **input** $u[k] := \begin{bmatrix} \dddot{c}[k] \end{bmatrix}$
- **Linear time-invariant system:** $x[k+1] = Ax[k] + Bu[k]$

Trajectory formulation

Looking at the first few steps:

$$x[1] = Ax[0] + Bu[0]$$
$$x[2] = A^2x[0] + [AB\ B][u[0]^T\ u[1]^T]^T$$
$$x[3] = A^3x[0] + [A^2B\ AB\ B][u[0]^T\ u[1]^T\ u[2]^T]^T$$

Define the state and input trajectory vectors:

$$X = \begin{bmatrix} x[0] \\ \vdots \\ x[N] \end{bmatrix} \qquad U = \begin{bmatrix} u[0] \\ \vdots \\ u[N-1] \end{bmatrix}$$

Then we have $X = \Phi x[0] + \Psi U$ with:

$$\Phi = \begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} \qquad \Psi = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ B & 0 & \cdots & 0 \\ AB & B & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix}$$

This is known as the *condensing algorithm* [BP84].

Trajectory constraints

Continuous-time system:

- **Dynamics:** $\ddot{c} = \omega^2(c - z) + g$
- **Constraints:** $Gz \leq h$

We can write equivalently:

- **Dynamics:** $z[k] = c[k] - \frac{\ddot{c}[k]}{\omega^2} = Z_k X = Z_k \Phi x[0] + Z_k \Psi U$
- **Feasibility:** $z_{\text{min}}[k] \leq z[k] \leq z_{\text{max}}[k]$

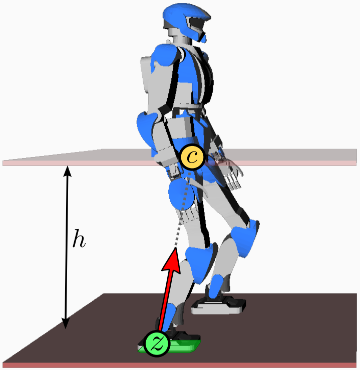
Our whole trajectory is linearly defined in U!

Continuous-time system:

- **Equation of motion:** $\ddot{c} = \omega^2 (c - z) + g$
- **Constraints:** $Gz \leq h$

Problems:

- **Controlability:** find $z(t)$ such that $c(t) \to c^*$
- **Optimal control:** find $z \in \arg\min_z \int_0^T \ell(c, \dot{c}, z) \mathrm{d}t$



Quadratic programming

A quadratic program can be generally written as:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \frac{1}{2} x^T P x + q^T x \\ \text{subject to} \quad & G x \leq h \\ & A x = b \\ & lb \leq x \leq ub \end{aligned}$$

In Python:

```python
from qpsolvers import solve_qp

M = np.array([[1., 2., 0.], [-8., 3., 2.], [0., 1., 1.]])
P = M.T @ M  # this is a positive definite matrix
q = np.array([3., 2., 3.]) @ M
G = np.array([[1., 2., 1.], [2., 0., 1.], [-1., 2., -1.]])
h = np.array([3., 2., -2.])

x = solve_qp(P, q, G, h, solver="proxqp")
```

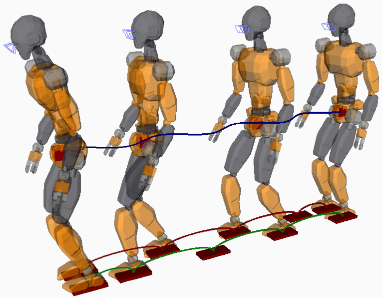
Setup: `pip install qpsolvers[open_source_solvers]`

Cost function

- Track desired ZMP reference
- Track desired CoM velocity
- Minimize CoM jerk

Constraints

- Transition: dynamics
- Feasibility: ZMP in support area
- (Viability: terminal DCM)



[9] Pierre-Brice Wieber. "Trajectory free linear model predictive control for stable walking in the presence of strong perturbations". In: *IEEE-RAS International Conference on Humanoid Robots*. 2006.

Linear OCP is a quadratic program

$$\min_{U} \quad w_z \sum_{k=1}^{N} \|z[k] - z^d[k]\|^2 + w_v \sum_{k=1}^{N} \|\dot{c}[k] - \dot{c}^d[k]\|^2 + w_j \sum_{k=1}^{N} \|u[k]\|^2$$

$$\text{s.t. } \forall k \quad c[k+1] = c[k] + T\dot{c}[k] + \frac{T^2}{2}\ddot{c}[k] + \frac{T^3}{6}u[k]$$

$$\dot{c}[k+1] = \dot{c}[k] + T\ddot{c}[k] + \frac{T^2}{2}u[k]$$

$$\ddot{c}[k+1] = \ddot{c}[k] + Tu[k]$$

Dynamics: $z[k] = c[k] - \frac{\ddot{c}[k]}{\omega^2}$

Feasibility: $z_{\text{min}}[k] \leq z[k] \leq z_{\text{max}}[k]$

$$(\text{Viability: } c[N] + \frac{\dot{c}[N]}{\omega} = \xi^d[N])$$

[10] Pierre-Brice Wieber. "Trajectory free linear model predictive control for stable walking in the presence of strong perturbations". In: *IEEE-RAS International Conference on Humanoid Robots*. 2006.

Quadratic programming for stair climbing

LIPM walking controller[1] = revisit Kajita *et al.* with QPs:

- **QP1:** linear model predictive control (ups [Kaj+03])
- **QP2:** wrench distribution (ups [Kaj+10])
- **QP3:** inverse kinematics (ups [Kaj+10])

Two consequences:

- Explicit cost functions
- Behavior switches on constraint saturation

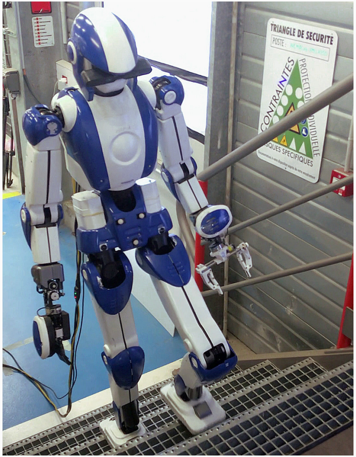
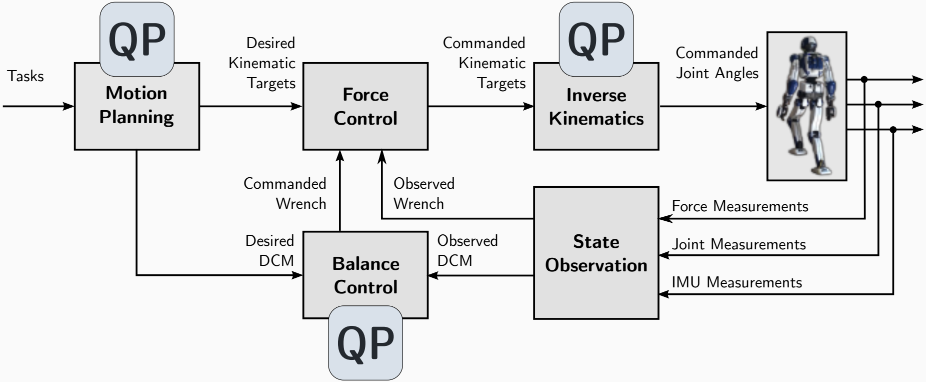


Figure 7: Stair climbing at Airbus

[11]Stéphane Caron, Abderrahmane Kheddar, and Olivier Tempier. “Stair Climbing Stabilization of the HRP-4 Humanoid Robot using Whole-body Admittance Control”. In: *IEEE International Conference on Robotics and Automation.* May 2019.

LIPM walking controller pipeline



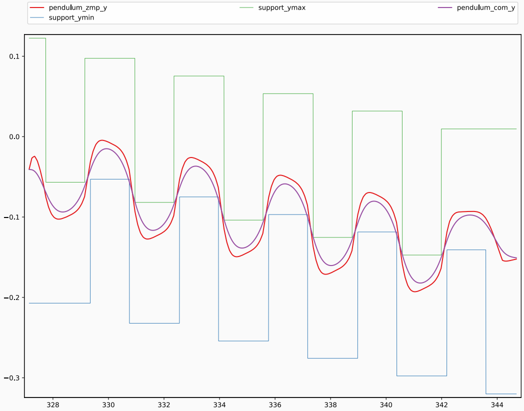


Figure 8: Horizontal axis for time, vertical axis for lateral coordinates $c_y, z_y, z_{\text{min}}, z_{\text{max}}$.

Lateral walking plan and measurements

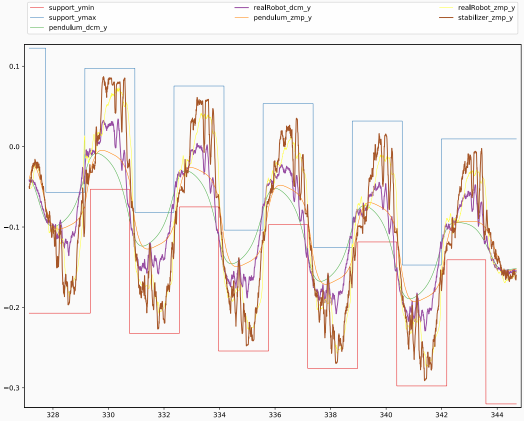
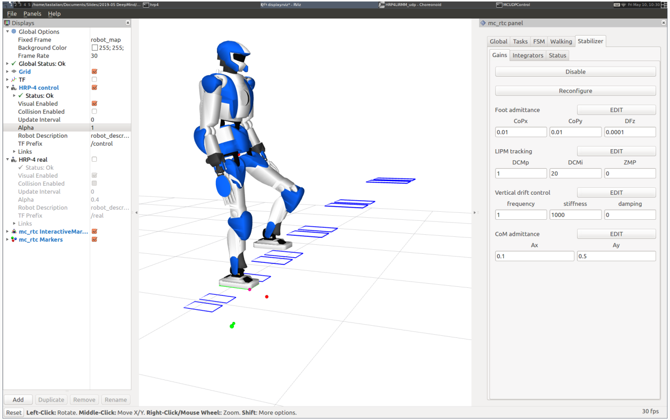


Figure 9: Horizontal axis for time, vertical axis for lateral coordinates $c_y, z_y, z_{\text{min}}, z_{\text{max}}$.

Try it out in a Docker!

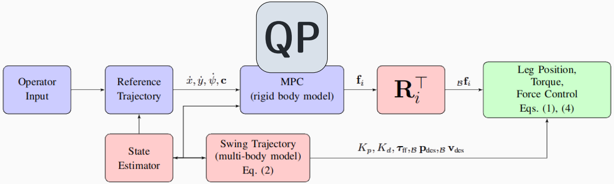



```
$ xhost +  # for X11 forwarding
$ docker run -it --rm --user ayumi -e DISPLAY=${DISPLAY} \
    -v /tmp/.X11-unix:/tmp/.X11-unix:rw \
    stephanecaron/lipm_walking_controller \
    lipm_walking --staircase
```

Linear OCP with centroidal dynamics

Figure 10: Humanoid, centroidal dynamics

Figure 11: Quadruped, centroidal dynamics



2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)
Madrid, Spain, October 1-5, 2018

Dynamic Locomotion in the MIT Cheetah 3 Through Convex Model-Predictive Control

Jared Di Carlo[1], Patrick M. Wensing[2], Benjamin Katz[3], Gerardo Bledt[1,3], and Sangbae Kim[3]

***Abstract*—This paper presents an implementation of model predictive control (MPC) to determine ground reaction forces for a torque-controlled quadruped robot. The robot dynamics are simplified to formulate the problem as convex optimization while still capturing the full 3D nature of the system. With the simplified model, ground reaction force planning problems are formulated for prediction horizons of up to 0.5 seconds, and are solved to optimality in under 1 ms at a rate of 20-30 Hz. Despite using a simplified model, the robot is capable of robust locomotion at a variety of speeds. Experimental results demonstrate control of gaits including stand, trot, flying-trot, pronk, bound, pace, a 3-legged gait, and a full 3D gallop. The robot achieved forward speeds of up to 3 m/s, lateral speeds up to 1 m/s, and angular speeds up to 180 deg/sec. Our approach is general enough to perform all these behaviors with the same set of gains and weights.**

Fig. 1. The MIT Cheetah 3 Robot galloping at 2.5 m/s

I. INTRODUCTION

Control of highly dynamic legged robots is a challenging problem due to the underactuation of the body during many gaits and due to constraints placed on ground reaction forces. As an example, during dynamic gaits[4] such as bounding or galloping, the body of the robot is always underactuated. Additionally, ground reaction forces must always remain in a friction cone to avoid slipping. Current solutions for highly dynamic locomotion include heuristic controllers for hopping and bounding [1], which are effective, but difficult to tune; two-dimensional planar simplifications [2], which are only applicable for gaits without lateral or roll dynamics; and evolutionary optimization for galloping [3], which cannot currently be solved fast enough for online use. Recent results on hardware include execution of bounding limit cycles discovered offline with HyQ [4] and learned pronking, trotting, and bounding gaits on StarlETH [5].

Predictive control can stabilize these dynamic gaits by anticipating periods of flight or underactuation, but is difficult to solve due to the nonlinear dynamics of legged robots and the large number of states and control inputs. Nonlinear optimization has been shown to be effective for predictive control of hopping robots [6], humanoids [7], [8], and quadrupeds [9], with [9] demonstrating the utility of heuristics to regularize the optimization. Another common approach is to use both a high-level planner, such as in [10], [11] and a lower level controller to track the plan. More recently, the experimental results in [12] show that whole-body nonlinear MPC can be used to stabilize trotting and jumping.

The stabilization of the quadruped robot HyQ using convex optimization discussed in [13] demonstrates the utility of convex optimization, but the approach cannot be immediately extended to dynamic gaits due to the quasi-static simplifications made to the robot model. Similarly, in bipedal locomotion, convex optimization has been used to find the best forces to satisfy instantaneous dynamics requirements [14] and to plan footsteps with the linear inverted pendulum model [15] but the latter approach does not include orientation in the predictive model.

While galloping is well studied in the field of biology [16], [17], surprisingly few hardware implementations of galloping exist. The first robot to demonstrate galloping was the underactuated quadruped robot Scout II [18], which reached 1.3 m/s, but had limited control of yaw. The MIT Cheetah 1 robot [19] achieved high-speed galloping, but was constrained to a plane. To the best of our knowledge, the only previous implementation of a fully 3D gallop with yaw control is on the hydraulically actuated WildCat robot [20], developed by Boston Dynamics. Unfortunately, no specific details about WildCat or its control system have been published.

The main contribution of this paper is a predictive controller which stabilizes a large number of gaits, including those with complex orientation dynamics. On hardware, we achieved a maximum yaw rate of 180 deg/sec and a maximum linear velocity of 3.0 m/s during a fully 3D gallop, which we believe to be the fastest gallop of an electrically actuated robot, and the fastest angular velocity of any legged robot similar in scale to Cheetah 3. Our controller can be formulated as a single convex optimization problem which considers a 3D, 12 DoF model of the robot. The solution of

Authors are with the [1]Department of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology, Cambridge, MA, 02139, USA; the [2]Department of Aerospace and Mechanical Engineering at the University of Notre Dame, Notre Dame, IN, 46556; and the [3]Department of Mechanical Engineering at the Massachusetts Institute of Technology, Cambridge, MA, 02139, USA. email: dicarloj@mit.edu, pwensing@nd.edu, benkatz@mit.edu, gbledt@mit.edu, sangbae@mit.edu

[4]In this paper, the term *dynamic gaits* is used to refer to gaits with significant periods of flight or underactuation.

This work was supported by the National Science Foundation [NSF-IIS-1350879] and the Air Force Office of Scientific Research [FA2386-17-1-4661]

978-1-5386-8094-0/18/$31.00 ©2018 IEEE 7440

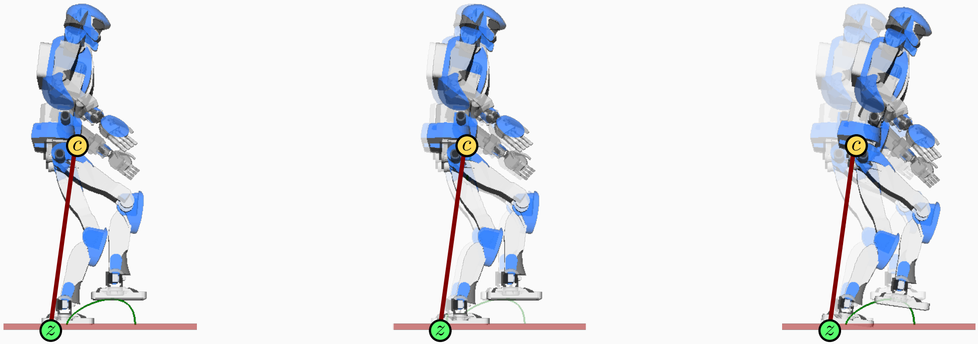
Linearize model predictive control QP:

- **Cost:** trajectory tracking + input regularization
- **Equality constraints:** no force on swing feet
- **Inequality constraints:** friction cones

[12] Jared Di Carlo, Patrick M Wensing, Benjamin Katz, Gerardo Bledt, and Sangbae Kim. "Dynamic locomotion in the mit cheetah 3 through convex model-predictive control". In: *IEEE/RSJ international conference on intelligent robots and systems*. 2018.

Balance control

Motivation

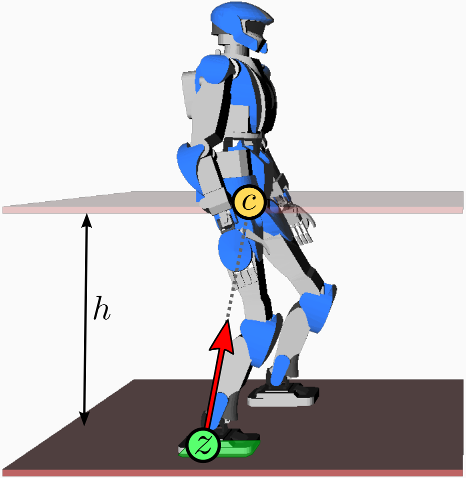


Plan$(t + \Delta t)$ Simulation$(t + \Delta t)$ Real$(t + \Delta t)$

Linear inverted pendulum mode

Assumptions:

- Rigid joints, sufficient power
- Conservation of angular momentum
- Constant CoM height



Equation of motion

$$\ddot{c} = \omega^2 (c - z) + g$$

- ω is a constant
- z: *zero-tilting moment point* (ZMP)
- In horiz. plane $+g$ is usually omitted

[13] Shuuji Kajita, Fumio Kanehiro, Kenji Kaneko, Kazuhito Yokoi, and Hirohisa Hirukawa. "The 3D Linear Inverted Pendulum Mode: A simple modeling for a biped walking pattern generation". In: *IEEE/RSJ International Conference on Intelligent Robots and Systems*. 2001.

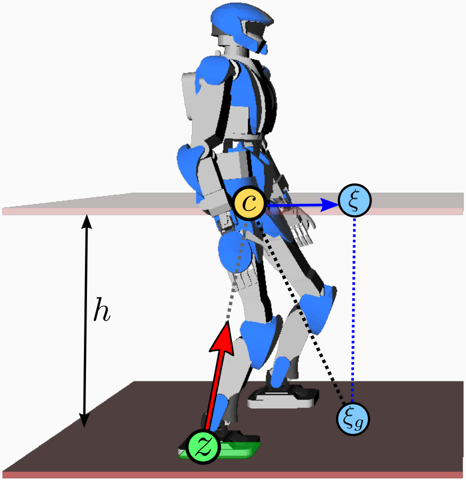
Divergent component of motion

- Linear inverted pendulum mode: $\ddot{c} = \omega^2(c - p)$
- Divergent component of motion: $\xi := c + \frac{\dot{c}}{\omega}$

DCM dynamics

$$\dot{\xi} = \omega(\xi - p)$$

- Point ξ_g where stepping will stop walking



Balance control with feet

- DCM dynamics:

$$\dot{\xi} = \omega(\xi - z)$$

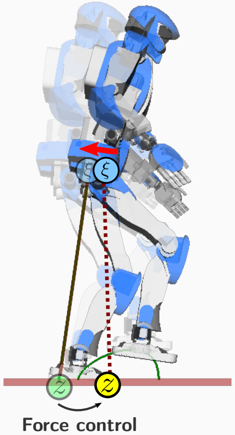
- Regulate the ZMP by **force control**:

$$z = z^d + \xi - k(\xi^d - \xi)$$

- Closed loop: $\xi \to \xi^d$

$$\dot{\xi} = k\omega(\xi^d - \xi)$$

- As long as the ZMP target is feasible...



- DCM dynamics:

$$\dot{\xi} = \omega(\xi - z)$$

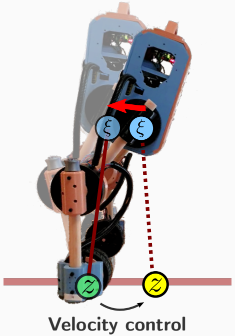
- Regulate the ZMP by **velocity control**:

$$z = z^d + \xi - k(\xi^d - \xi)$$

- Closed loop: $\xi \to \xi^d$

$$\dot{\xi} = k\omega(\xi^d - \xi)$$

- As long as the ZMP target is feasible...



What did we see?

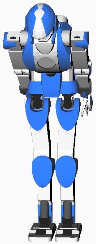
What did we see?

Balancing and walking legged robots:

- **Physics:** linear inverted pendulum, ZMP, support area
- **Optimal control:** LTI system, OC as a quadratic program
- **Balancing:** adjust ZMP based on a DCM of the CoM

Applies to humanoids, quadrupeds, wheeled bipeds, flywheeled coffee makers, etc.

That’s all folks!



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