# Walking and stair climbing controller for locomotion in an aircraft factory by the HRP-4 humanoid robot

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# MOTOR INTELLIGENCE

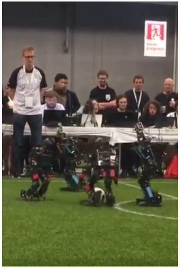
Chess :

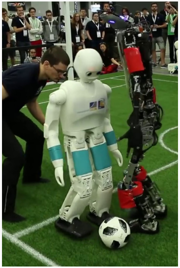
- **1956 :** simplified rules, beats novice
- **1967 :** full rules, wins tournament
- **1981 :** beats master in tournament
- **1997 :** beats world champion


**Robot Soccer World Cup (Robocup)**

« ... to develop a team of humanoid robots that is able to win against the official human World Soccer Champion team until 2050. »

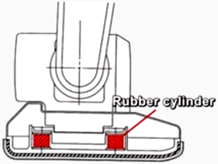
- Established in 1996
- Still **simplified** rules in 2019
- Yearly update towards human rules

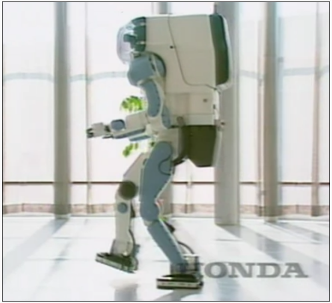




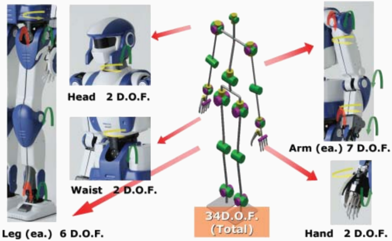
Public demonstration in 1998 :

- Zero-tilting Moment Point (ZMP) control
- Ground reaction force control
- Impact absorption (SEA before SEA) :

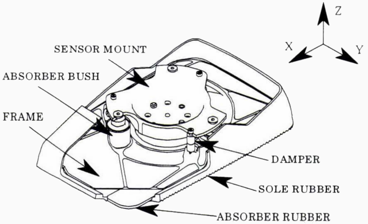



1. Kazuo HIRAI, Masato HIROSE, Yuji HAIKAWA et Toru TAKENAKA. « The development of Honda humanoid robot ». In : *IEEE International Conference on Robotics and Automation*. 1998.




Stiff position control on all joints




Mechanical flexibility at the ankles

2. Kenji KANEKO, Fumio KANEHIRO, Mitsuharu MORISAWA, Kazuhiko AKACHI, Gou MIYAMORI, Atsushi HAYASHI et Noriyuki KANEHIRA. « Humanoid robot HRP-4 - Humanoid Robotics Platform with Lightweight and Slim Body ». In : *IEEE/RSJ International Conf. on Intelligent Robots and Systems*. 2011.

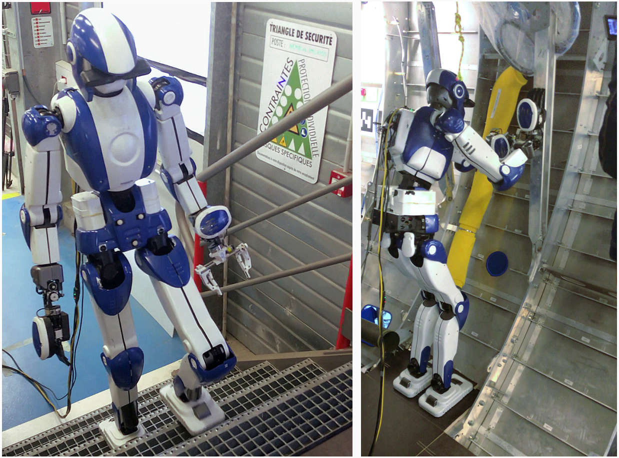
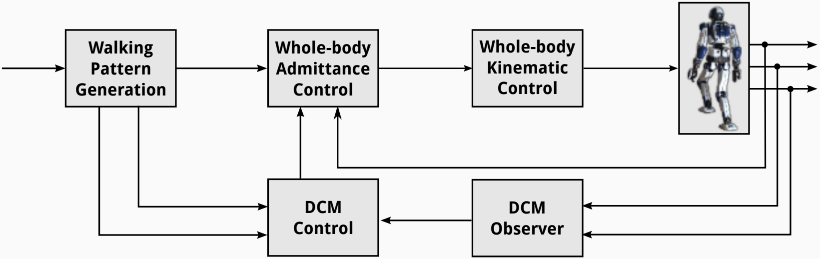



FIGURE 1 : Locomotion, balancing and manipulation to achieve the use case



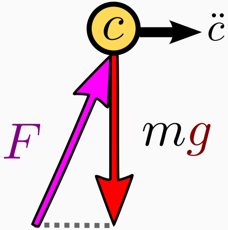

Source code : `https://github.com/stephane-caron/lipm_walking_controller/`

# PHYSICS : FROM SIMPLE TO COMPLEX

**Newton's second law**

$$m\ddot{c} = mg + F$$

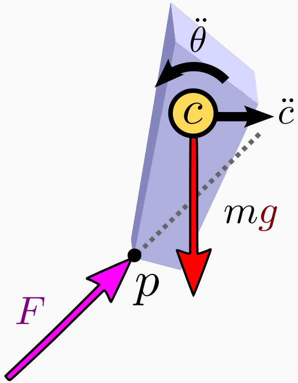
- $m$ : total mass
- $c$ : center of mass (CoM)
- $g$ : acceleration due to gravity
- $F$ : external force



## Newton-Euler equations (2D)

$$m\ddot{c} = mg + F$$
$$I\ddot{\theta} = (p - c) \times F$$

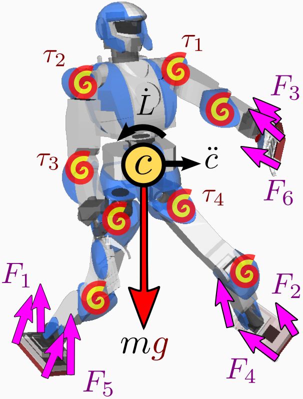
- $I$ : moment of inertia around the CoM
- $\dot{\theta}$ : angular velocity around the CoM
- $p$ : contact point
- $F$ : external force



## Equation of motion

$$M\ddot{q} = G + S^T\tau + J^T F$$

- $n$ : number of actuated joints
- $q$ : generalized coordinates $(n + 6)$
- $M$ : inertia matrix $(n + 6)^2$
- $G$ : gravity and nonlinear effects
- $\tau$ : actuated joint torques
- $J$ : contact Jacobian
- $F$ : external forces



# CONTROL : FROM COMPLEX TO SIMPLE

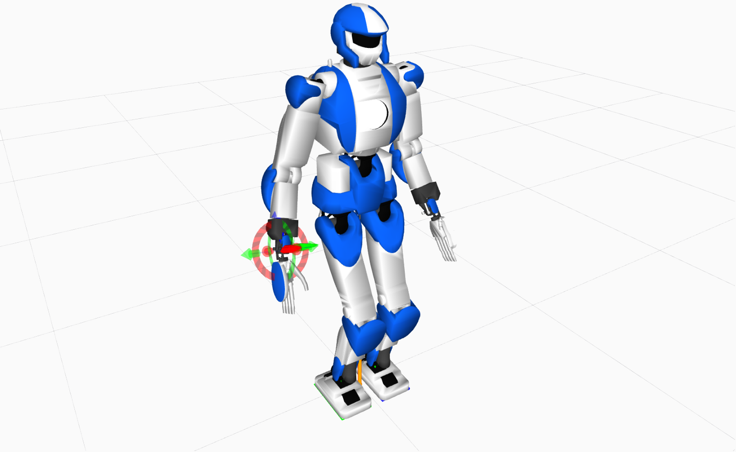


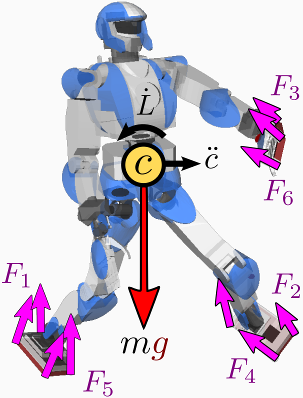
FIGURE 2 : Control *task* targets rather than generalized coordinates

If motors can produce $\tau \in \mathbb{R}^n$, equation of motion reduces to Newton-Euler again :

**Equation of motion**

$$m\ddot{c} = mg + \sum_i F_i$$
$$\dot{L}_c = \sum_i (p_i - c) \times F_i$$

- $L_c$ : angular momentum around $c$
- $p_i$ : application point of force $F_i$




3. David E. Orin, Ambarish Goswami et Sung-Hee Lee. « Centroidal dynamics of a humanoid robot ». In : *Autonomous Robots* 35.2 (oct. 2013).

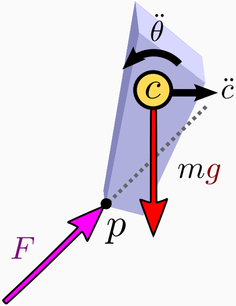



FIGURE 3 : Net contact force does not go through CoM $\Rightarrow \dot{L} = I\ddot{\theta} > 0$, body rotates and translates

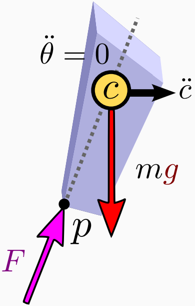



FIGURE 4 : Net contact force goes through CoM $\Rightarrow \dot{L} = 0$, body translates only, no rotation

**Bottom line**

A constant angular momentum reduces the system to translation

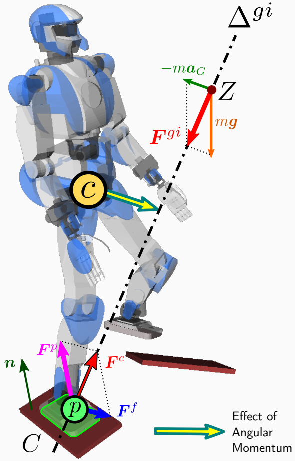
**Center of pressure (CoP)**

Point $C$ on the contact surface where the resultant of *pressure* forces $F^p$ is applied.

**Zero-tilting Moment Point (ZMP)**

Points $Z$ where the moment of the contact wrench is aligned with the contact normal $n$.

- Informally : the ZMP is the point where the net contact force is applied.
- Formally : the ZMP axis intersects the contact surface at the CoP.



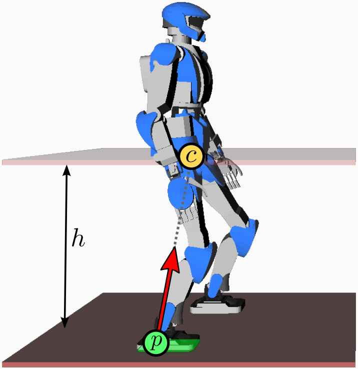

4. P. Sardain et G. Bessonnet. « Forces acting on a biped robot. center of pressure-zero moment point ». In : *IEEE Transactions on Systems, Man and Cybernetics, Part A : Systems and Humans* 34.5 (2004).

- Constant angular momentum $\dot{L}_c = 0$
- Constant CoM height $c^z = h$

**Equation of motion**

$$\ddot{c} = \omega^2 (c - p)$$

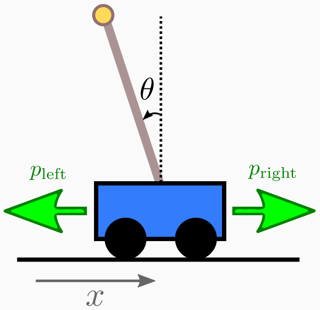
- $\omega^2 = g/h$ is a constant
- $p$ : *zero-tilting moment point* (ZMP)




5. Shuuji KAJITA, Fumio KANEHIRO, Kenji KANEKO, Kazuhito YOKOI et Hirohisa HIRUKAWA. « The 3D Linear Inverted Pendulum Mode : A simple modeling for a biped walking pattern generation ». In : *IEEE/RSJ International Conference on Intelligent Robots and Systems*. 2001.

**Equation of motion**

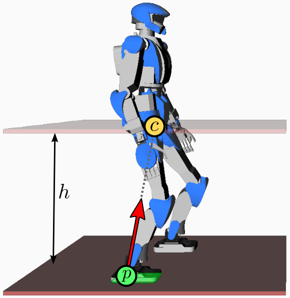
$$\ddot{\theta} \approx \omega^2(\theta - p)$$




- $p \propto \ddot{x}$ is a discrete action
- $x$ is unconstrained
- $\theta$ may go down to $\pm\pi$

**Equation of motion**

$$\ddot{c} = \omega^2(c - p)$$



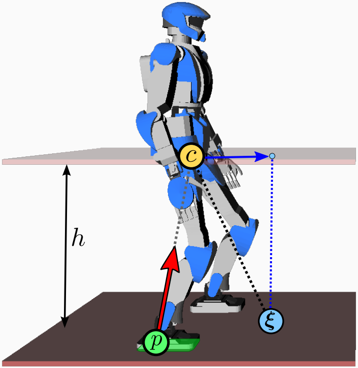

- $p$ is a hybrid continuous action
- $p$ is constrained to the foot sole
- $c$ may diverge to $\pm\infty$

- Linear inverted pendulum mode : $\ddot{c} = \omega^2(c - p)$
- Divergent component of motion : $\xi := c + \frac{\dot{c}}{\omega}$

**Equation of motion**

$$\dot{\xi} = \omega(\xi - p)$$

- Maximizes basin of attraction among linear feedback controllers [Sug09]
- Boundedness condition [LHM14]




6. Toru TAKENAKA, Takashi MATSUMOTO et Takahide YOSHIIKE. « Real time motion generation and control for biped robot-1st report : Walking gait pattern generation ». In : *IEEE/RSJ International Conference on Intelligent Robots and Systems*. 2009.

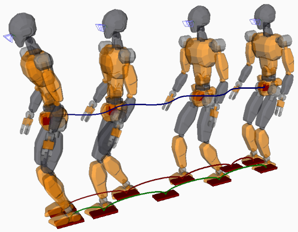
# Walking pattern generation

## Cost function

- Track desired ZMP reference
- Track desired CoM velocity
- Minimize CoM jerk

## Constraints

- Consistency : equation of motion
- Feasibility : ZMP in support area
- Viability : terminal DCM



7. Pierre-Brice WIEBER. « Trajectory free linear model predictive control for stable walking in the presence of strong perturbations ». In : *IEEE-RAS International Conference on Humanoid Robots*. 2006.

$$\min_{\dddot{c}[1\ldots N]} \quad w_z \sum_{k=1}^{N} \|p[k] - p^d[k]\|^2 + w_v \sum_{k=1}^{N} \|\dot{c}[k] - \dot{c}^d[k]\|^2 + w_j \sum_{k=1}^{N} \|\dddot{c}[k]\|^2$$

$$\text{s.t. } \forall k \quad c[k+1] = c[k] + T\dot{c}[k] + \frac{T^2}{2}\ddot{c}[k] + \frac{T^3}{6}\dddot{c}[k]$$

$$\dot{c}[k+1] = \dot{c}[k] + T\ddot{c}[k] + \frac{T^2}{2}\dddot{c}[k]$$

$$\ddot{c}[k+1] = \ddot{c}[k] + T\dddot{c}[k]$$

**Equation of motion :** $p[k] = c[k] - \frac{\ddot{c}[k]}{\omega^2}$

**Feasibility :** $p_{\min}[k] \leq p[k] \leq p_{\max}[k]$

**Viability :** $c[N] + \frac{\dot{c}[N]}{\omega} = \xi^d[N]$

8. Pierre-Brice Wieber. « Trajectory free linear model predictive control for stable walking in the presence of strong perturbations ». In : *IEEE-RAS International Conference on Humanoid Robots*. 2006.

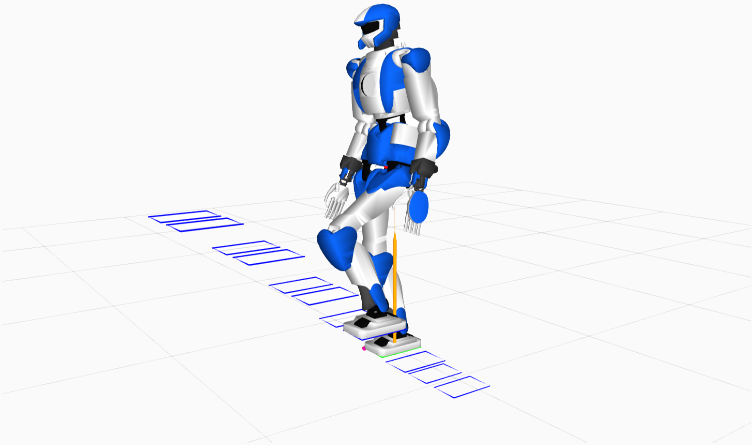
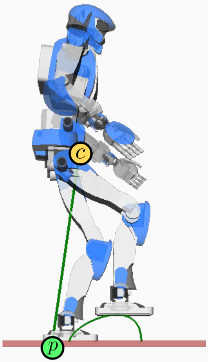


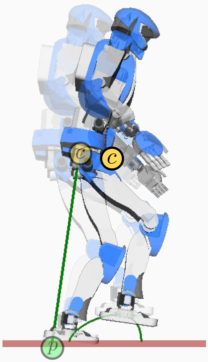
FIGURE 5 : Stair climbing motion in mc_rtc

# Walking stabilization

Actuated joints converge but unactuated floating base *diverges* :




Planned motion




On robot without stabilization

# VISUALIZATION

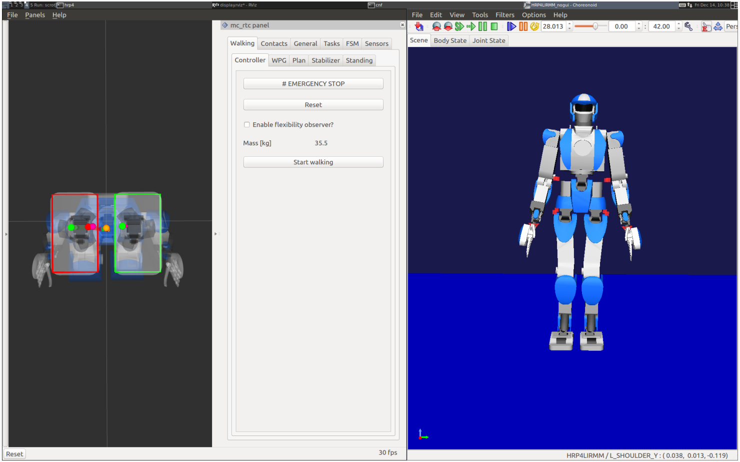



FIGURE 6 : Standing stabilization under external forces

- The floating base is unactuated
- We can control it via the CoM and Newton-Euler equations
- In the LIPM, they are reduced to :

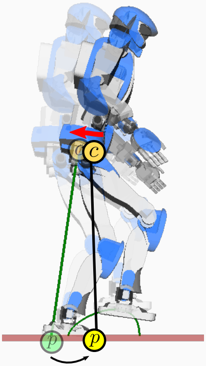
$$\ddot{c} = \omega^2 (c - p)$$

- Feedback is realized by **indirect force control** of the ZMP :

$$p = p^d - k_p (c^d - c) - k_d (\dot{c}^d - \dot{c})$$

- Best control is by DCM feedback :

$$p = p^d - k(\xi^d - \xi)$$



... but our robot is **position-controlled** ?

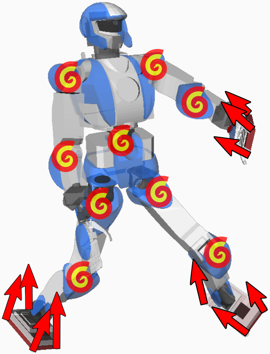
Split control into two components :

**Admittance control**

Change position targets in order to track desired forces

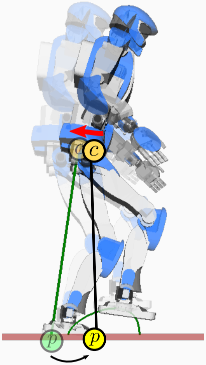
**DCM feedback control**

Assuming force control, decide reaction forces that drive the floating base



Admittance control strategies for different components of the net contact wrench :

- CoP at each contact [Kaj+01b]
- Pressure distribution [Kaj+10]
- CoM admittance control [Nag99]



- Rotate end-effector to move its CoP
- Assumes compliance at contact :

$$\tau = K_e(\theta - \theta_e)$$

- Apply damping control :

$$\dot{\theta} = A_{cop}(\tau_d - \tau)$$

- Closed-loop behavior has $\tau \to \tau_d$

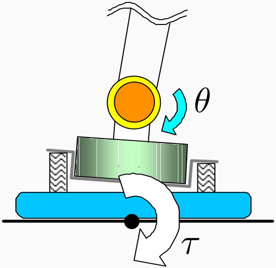



Figure adapted from [Kaj+01b]

9. Shuuji Kajita, Kazuhito Yokoi, Muneharu Saigo et Kazuo Tanie. « Balancing a Humanoid Robot Using Backdrive Concerned Torque Control and Direct Angular Momentum Feedback ». In : *IEEE International Conference on Robotics and Automation*. 2001.

- Net vertical force compensates gravity $\Rightarrow$ only need to control :

$$\Delta f_z = f_{Rz} - f_{Lz}$$

- Push down with foot that needs more pressure, lift the other one
- Apply damping control :

$$\dot{z}_{ctrl} = A_z(\Delta f_{zd} - \Delta f_z)$$

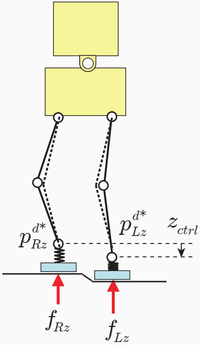


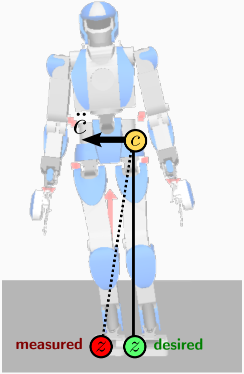

Figure adapted from [Kaj+10]

10. Shuuji Kajita, Mitsuharu Morisawa, Kanako Miura, Shin'ichiro Nakaoka, Kensuke Harada, Kenji Kaneko, Fumio Kanehiro et Kazuhito Yokoi. « Biped walking stabilization based on linear inverted pendulum tracking ». In : *IEEE/RSJ International Conference on Intelligent Robots and Systems*. 2010.

- Accelerate CoM against ZMP error :

$$\ddot{c} = A_c(p - p_d)$$

- Amounts to *translational* hip strategy
- **Counterintuitive :** if you fall forward, accelerate forward !




11. Ken'ichiro NAGASAKA. « Whole-body Motion Generation for a Humanoid Robot by Dynamics Filters ». In : *PhD thesis* (1999). The University of Tokyo, in Japanese.

Which ones to choose ?

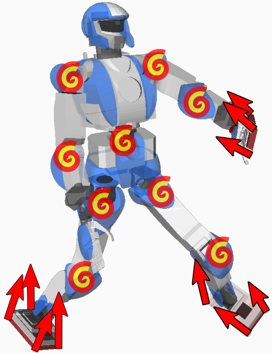
**End-effector strategies**

- CoP at each contact [Kaj+01b]
- Pressure distribution [Kaj+10]

... are sufficient to control the net wrench, yet :

**CoM admittance control [Nag99]**

- uses other joints, *e.g.* hips
- helps recover from ZMP saturation



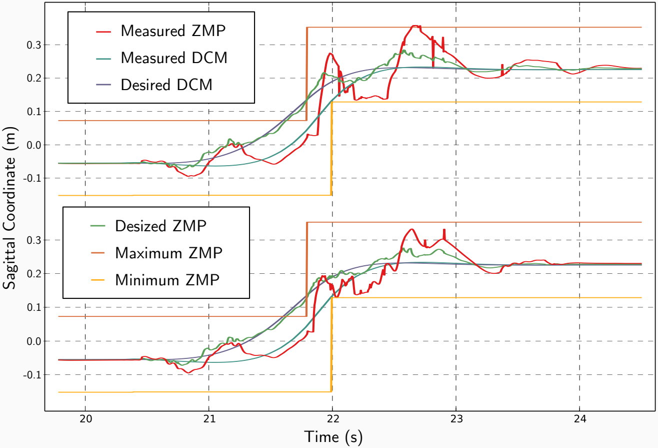
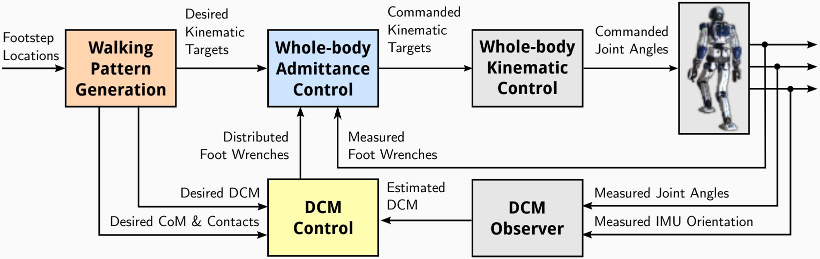
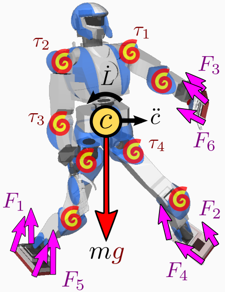


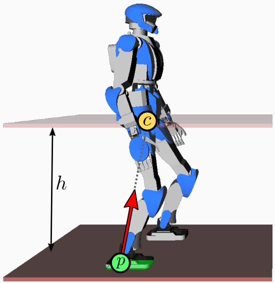

**FIGURE 7 : Top :** no CoM admittance control. **Bottom :** with $A_c = 20$ [Hz$^2$].



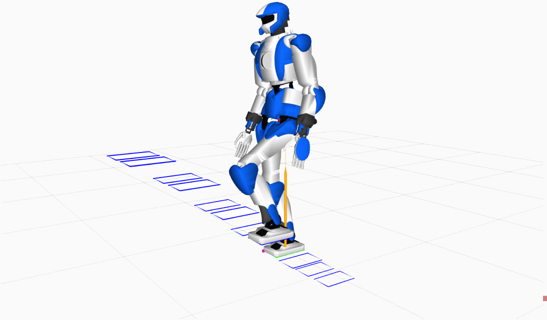

Source code : `https://github.com/stephane-caron/lipm_walking_controller/`

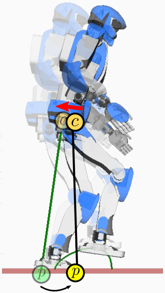
## WHAT HAVE WE SEEN ?




- **Physics :** from a simple to complex system
- **Control :** distribute complexity, simple high level





- **Body :** task function approach
- **Spine :** stabilization by DCM feedback
- **Brain :** model predictive control

Thank you for your attention!

[Hir+98] Kazuo HIRAI, Masato HIROSE, Yuji HAIKAWA et Toru TAKENAKA. « The development of Honda humanoid robot ». In : *IEEE International Conference on Robotics and Automation*. 1998.

[Kaj+01a] Shuuji KAJITA, Fumio KANEHIRO, Kenji KANEKO, Kazuhito YOKOI et Hirohisa HIRUKAWA. « The 3D Linear Inverted Pendulum Mode : A simple modeling for a biped walking pattern generation ». In : *IEEE/RSJ International Conference on Intelligent Robots and Systems*. 2001.

[Kaj+01b] Shuuji KAJITA, Kazuhito YOKOI, Muneharu SAIGO et Kazuo TANIE. « Balancing a Humanoid Robot Using Backdrive Concerned Torque Control and Direct Angular Momentum Feedback ». In : *IEEE International Conference on Robotics and Automation*. 2001.

[Kaj+10] Shuuji KAJITA, Mitsuharu MORISAWA, Kanako MIURA, Shin'ichiro NAKAOKA, Kensuke HARADA, Kenji KANEKO, Fumio KANEHIRO et Kazuhito YOKOI. « Biped walking stabilization based on linear inverted pendulum tracking ». In : *IEEE/RSJ International Conference on Intelligent Robots and Systems*. 2010.

[Kan+11] Kenji KANEKO, Fumio KANEHIRO, Mitsuharu MORISAWA, Kazuhiko AKACHI, Gou MIYAMORI, Atsushi HAYASHI et Noriyuki KANEHIRA. « Humanoid robot HRP-4 - Humanoid Robotics Platform with Lightweight and Slim Body ». In : *IEEE/RSJ International Conf. on Intelligent Robots and Systems*. 2011.

[LHM14] Leonardo LANARI, Seth HUTCHINSON et Luca MARCHIONNI. « Boundedness issues in planning of locomotion trajectories for biped robots ». In : *IEEE-RAS International Conference on Humanoid Robots*. 2014.

[Nag99] Ken'ichiro NAGASAKA. « Whole-body Motion Generation for a Humanoid Robot by Dynamics Filters ». In : *PhD thesis* (1999). The University of Tokyo, in Japanese.

[OGL13] David E. ORIN, Ambarish GOSWAMI et Sung-Hee LEE. « Centroidal dynamics of a humanoid robot ». In : *Autonomous Robots* 35.2 (oct. 2013).

[SB04] P. SARDAIN et G. BESSONNET. « Forces acting on a biped robot. center of pressure-zero moment point ». In : *IEEE Transactions on Systems, Man and Cybernetics, Part A : Systems and Humans* 34.5 (2004).

[Sug09] Tomomichi SUGIHARA. « Standing stabilizability and stepping maneuver in planar bipedalism based on the best COM-ZMP regulator ». In : *IEEE International Conference on Robotics and Automation*. 2009.

[TMY09] Toru TAKENAKA, Takashi MATSUMOTO et Takahide YOSHIIKE. « Real time motion generation and control for biped robot-1st report : Walking gait pattern generation ». In : *IEEE/RSJ International Conference on Intelligent Robots and Systems*. 2009.

[Wie06] Pierre-Brice WIEBER. « Trajectory free linear model predictive control for stable walking in the presence of strong perturbations ». In : *IEEE-RAS International Conference on Humanoid Robots*. 2006.