ZMP SUPPORT AREAS FOR MULTI-CONTACT LOCOMOTION

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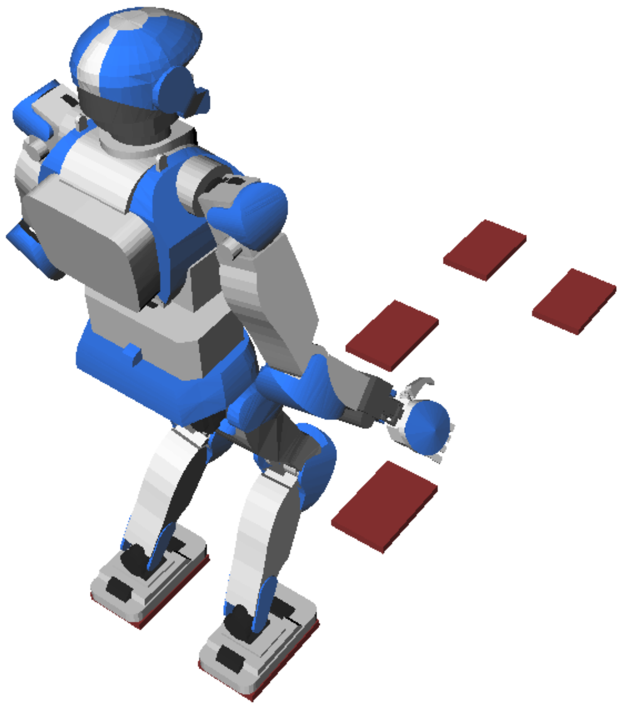
WALKING ON FLAT FLOORS



- Start from a footstep plan

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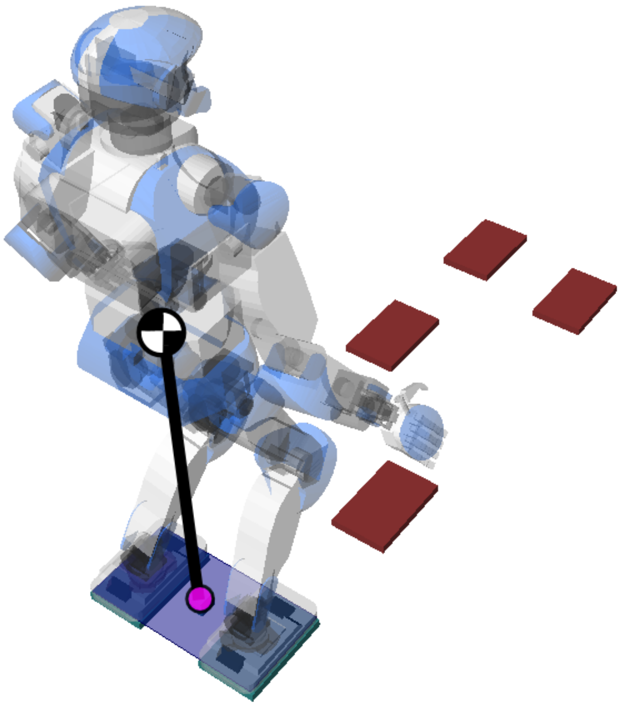

- Start from a footstep plan
- Regulate system dynamics around the **<u>Linear Inverted Pendulum Mode</u>** (LIPM)

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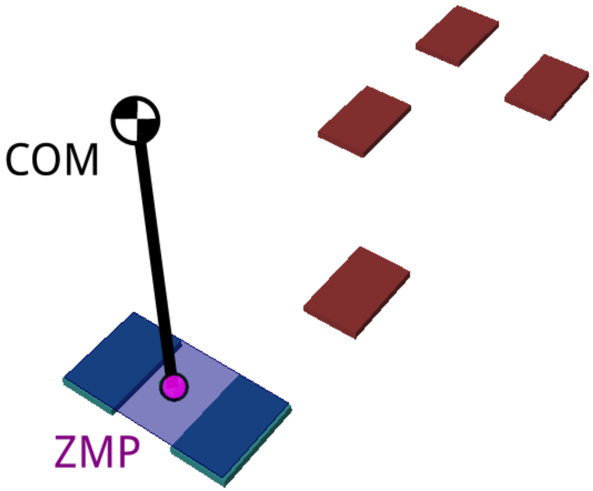

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  - **Contact stability criterion:** ZMP inside the support polygon

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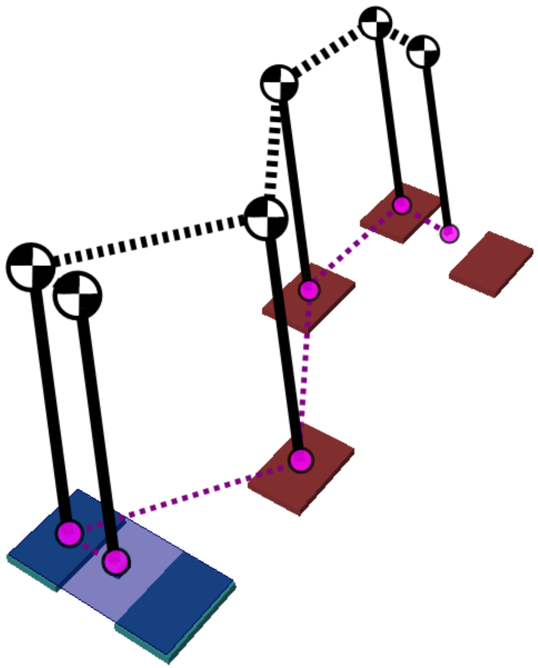

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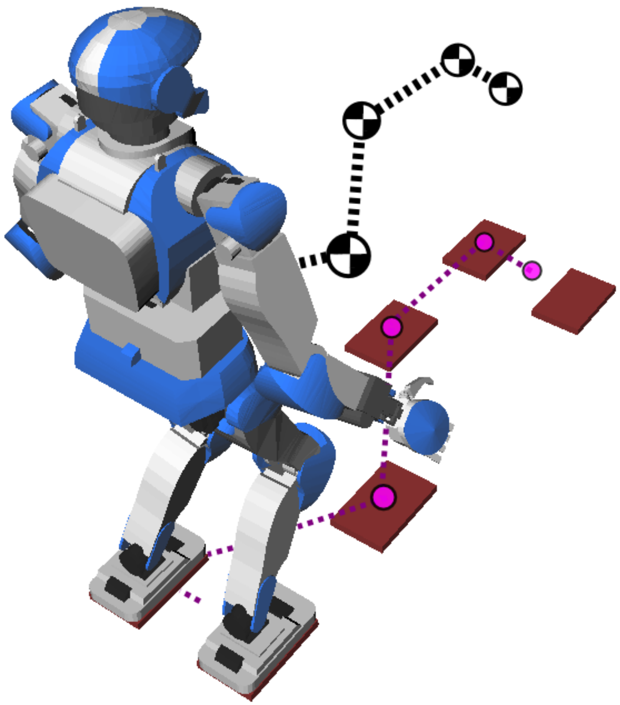

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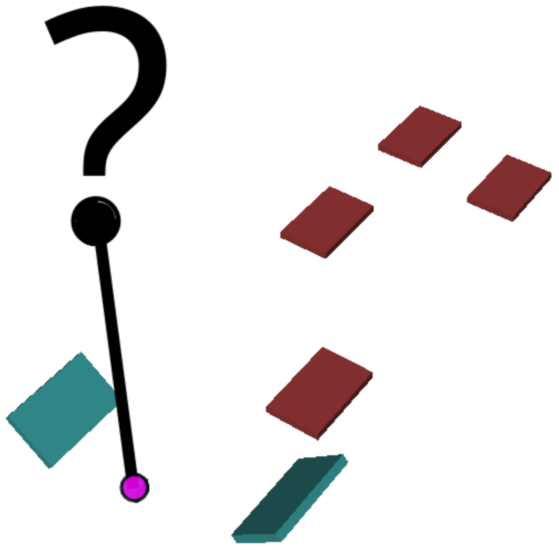
- Plan a trajectory for the pendulum
- Send it as reference to a whole-body controller

QUESTION



ZERO-TILTING MOMENT POINT

Definition

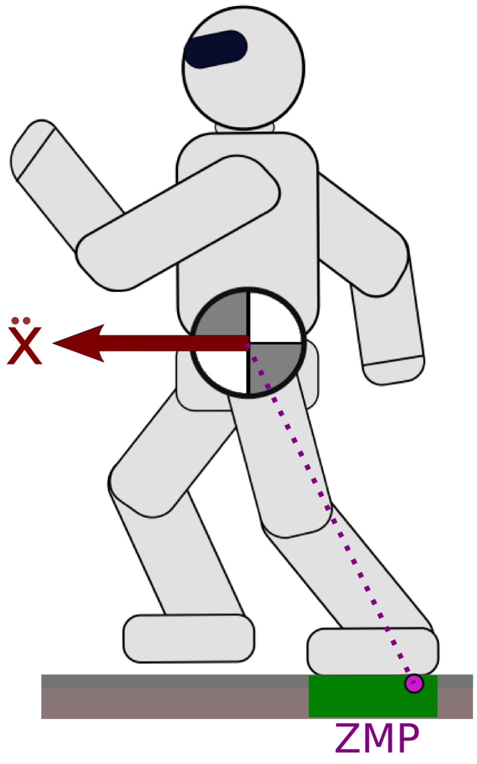
The ZMP is traditionally defined as the point *on the floor* where the moment of the contact wrench is parallel to the surface normal **(Sardain & Bessonnet, 2004)**.

Intuition

The point at which the robot “applies its weight”.

Contact stability

The ZMP must lie inside the support polygon (convex hull of ground contact points).[1]



LIMITATIONS

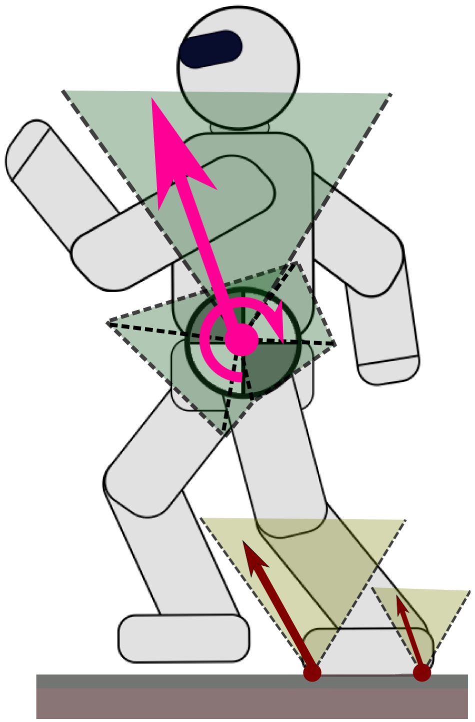
- This definition requires a single “floor” surface (no multi-contact)
- The support polygon does not account for frictional limits (slippage, yaw rotations).

CONTACT STABILITY

How to receive forces from the environment?

Weak Contact Stability

A motion or wrench is *weak contact stable* iff it can be realized by contact forces inside their friction cones.



Contact Wrench Cone

Friction cones can be combined as Contact Wrench Cone (CWC) at the COM—see e.g. **(Caron et al., 2015)**.
By construction,

$$w \in \mathrm{CWC} \Leftrightarrow \exists \{f_i \in \mathrm{FC}_i\}, \oplus f_i = w.$$

ZMP OF A WRENCH

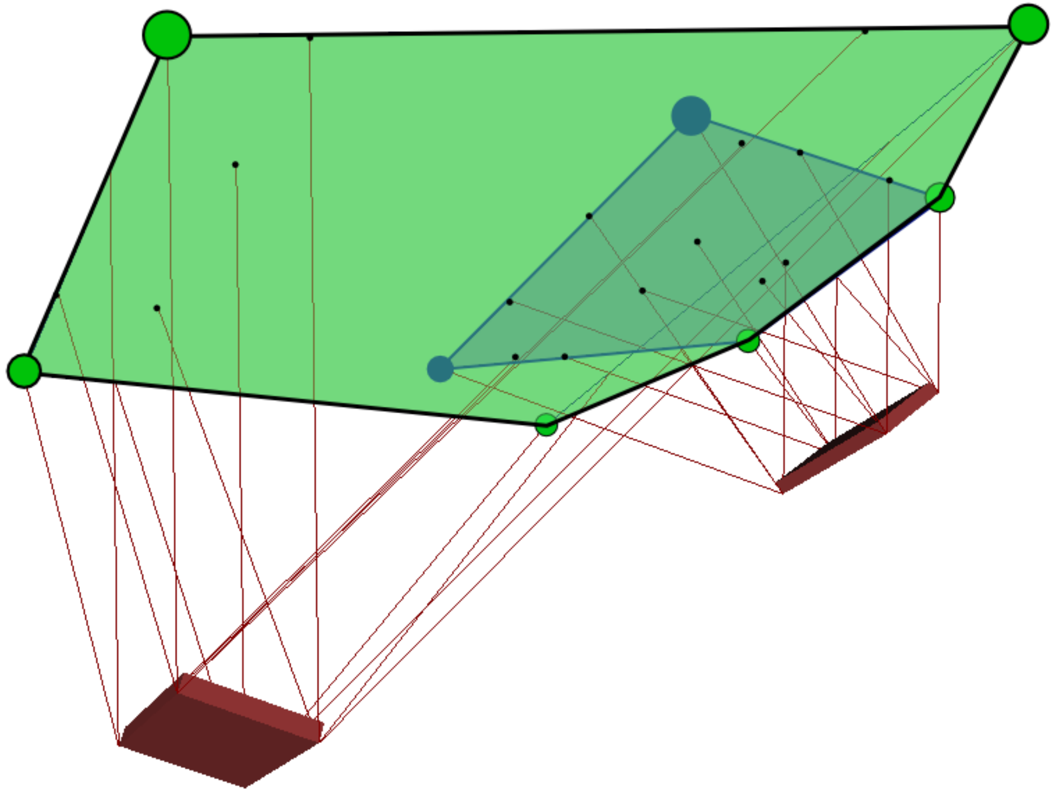
The ZMP is mathematically defined from a wrench **(Sardain & Bessonnet, 2004)**. The ZMP in the plane $\Pi(O, n)$ of normal n containing O is the point such that $n \times \tau_Z = 0$:

$$x_Z = \frac{n \times \tau_O}{n \cdot f} + x_O.$$

We define the *full support area* $\mathcal{S}$ as the image of the CWC by this equation.

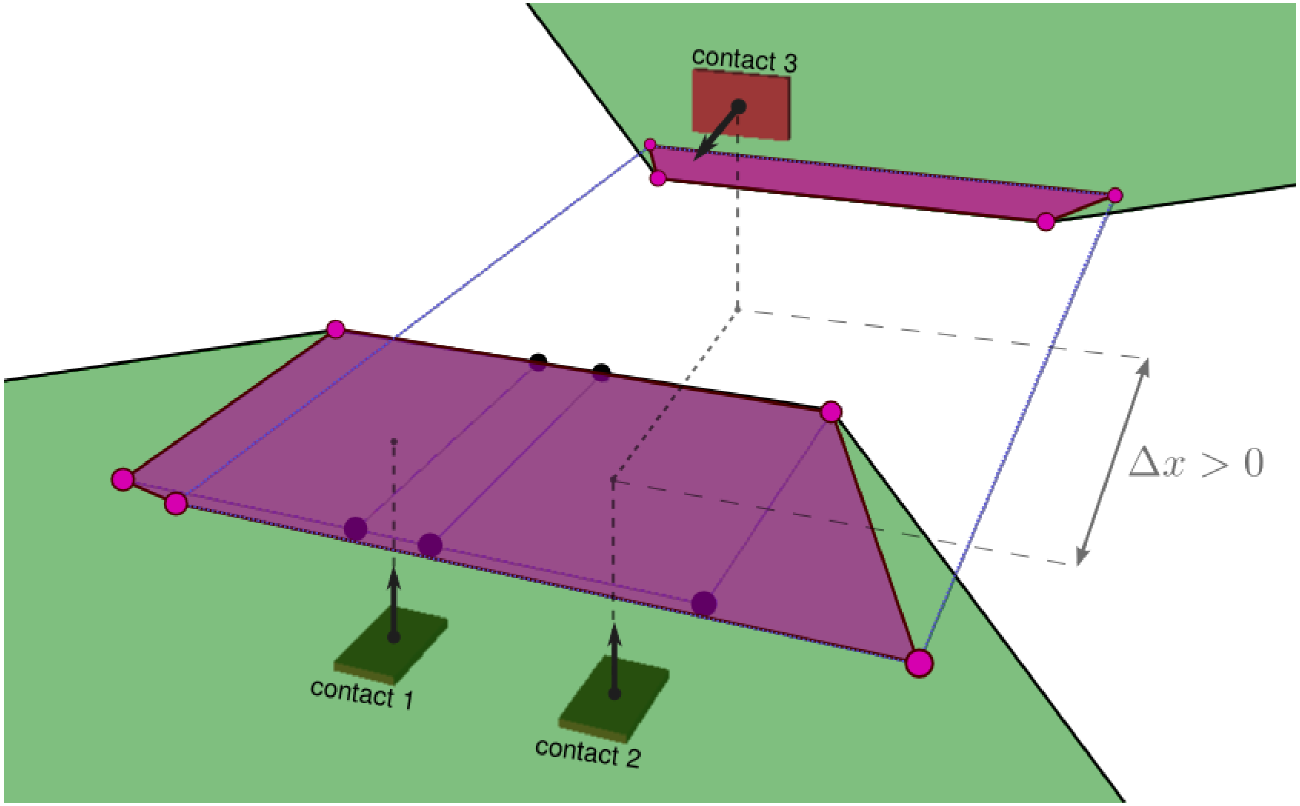
CONTRIBUTION 1

Good news! The image of the CWC can be computed geometrically:



CONTRIBUTION 1

Bad news! It is not always a polygon:



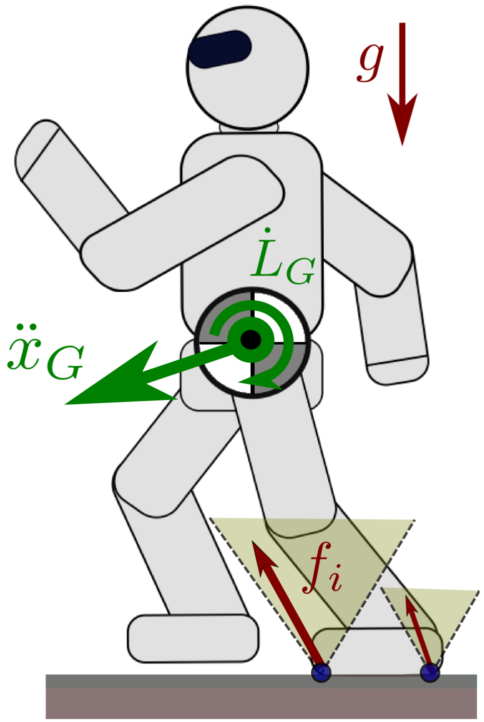
CENTROIDAL DYNAMICS

The **Newton-Euler equations** of the system are:

$$\begin{bmatrix} m\ddot{x}_G \\ \dot{L}_G \end{bmatrix} = \begin{bmatrix} mg \\ 0 \end{bmatrix} + \sum_{\text{contact } i} \begin{bmatrix} f_i \\ \overrightarrow{GC}_i \times f_i \end{bmatrix}$$

- m and g: total mass and gravity vector
- $\ddot{x}_G$: acceleration of the **center of mass** (COM)
- $\dot{L}_G$: rate of change of the **angular momentum**
- f_i: contact force received at contact point C_i

They show how the motion of unactuated DOFs results from interactions with the environment.



LINEAR PENDULUM MODE

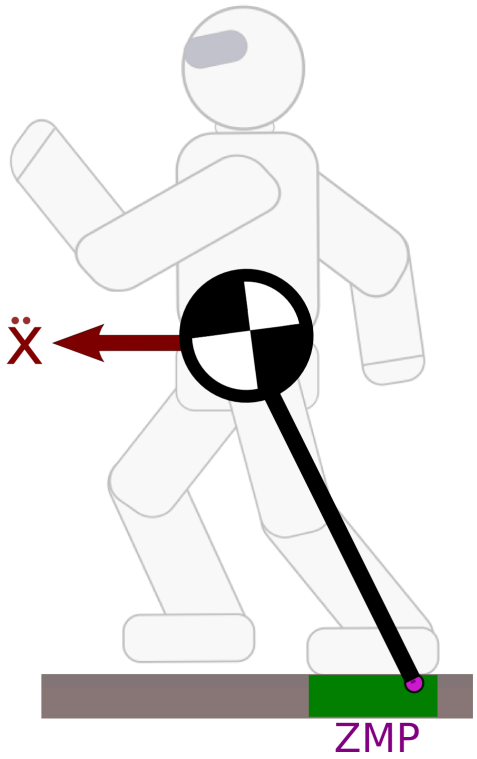
The Newton equation can be written equivalently:

$$\ddot{x}_G = \frac{g + \ddot{z}_G}{z_G - z_Z}(x_G - x_Z) - \frac{\dot{L}_{Gx}}{m(z_G - z_Z)}$$

where x now denotes X-Y plane coordinates. The Linear Inverted Pendulum Mode **(Kajita et al., 2001)** is obtained by constraining:

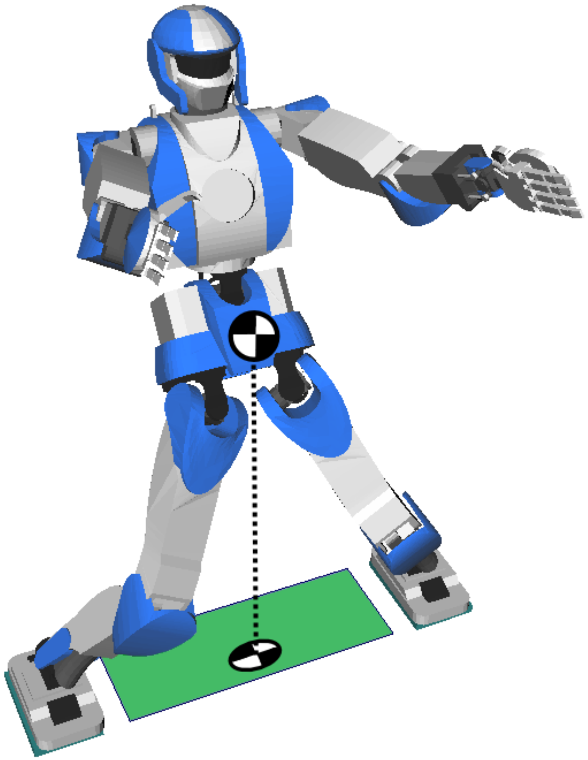
$$\begin{aligned} z_G - z_Z &= h \\ \dot{L}_G &= 0 \end{aligned}$$

The system dynamics become $\ddot{x}_G = \frac{g}{h}(x_Z - x_G)$.



OBSERVATION

The support area in the LIPM is *smaller* than the convex hull of contact points:



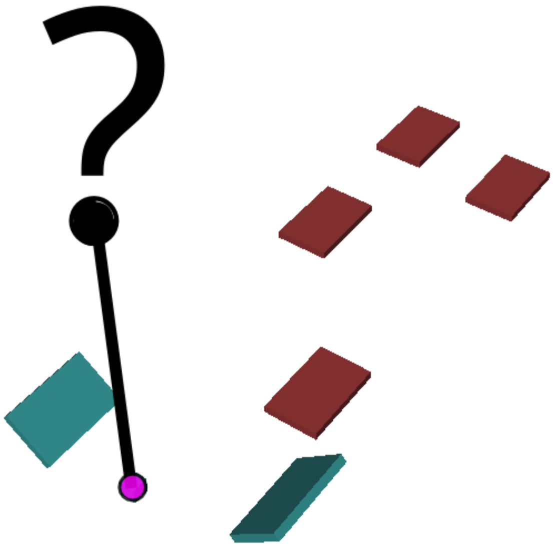
CONTRIBUTION 2

We provide an algorithm to compute the support area corresponding to the system:

$$\begin{aligned} w &\in \mathrm{CWC} \\ z_G - z_Z &= h \\ \dot{L}_G &= 0 \end{aligned}$$

We call it the *pendular support area*.

PENDULAR SUPPORT AREA



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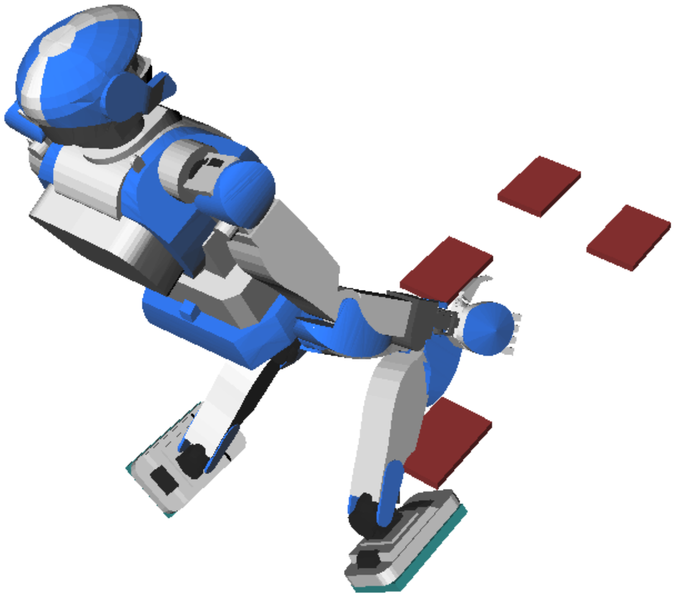

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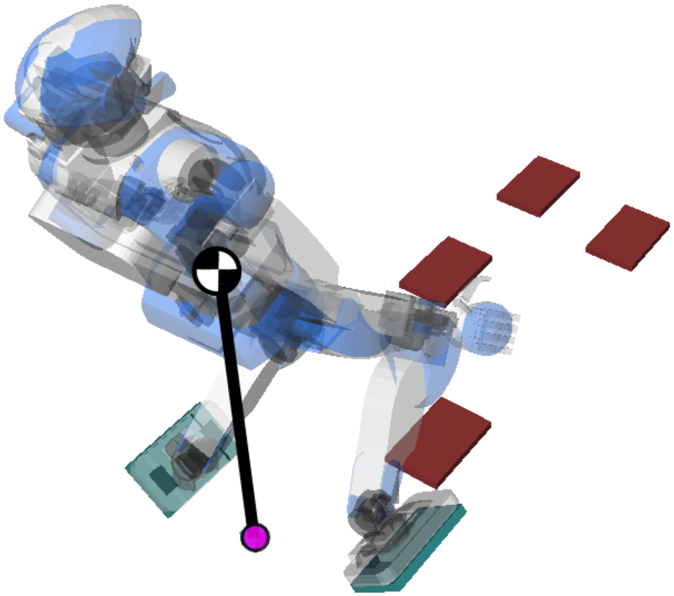

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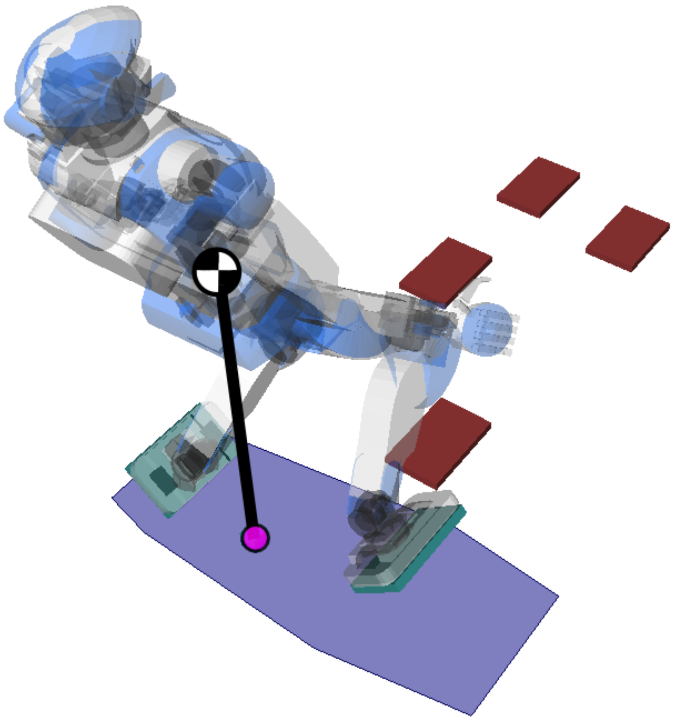

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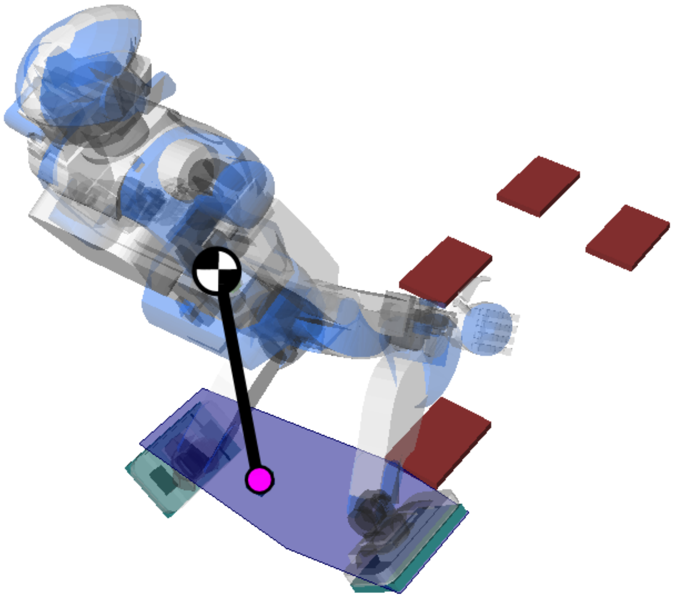

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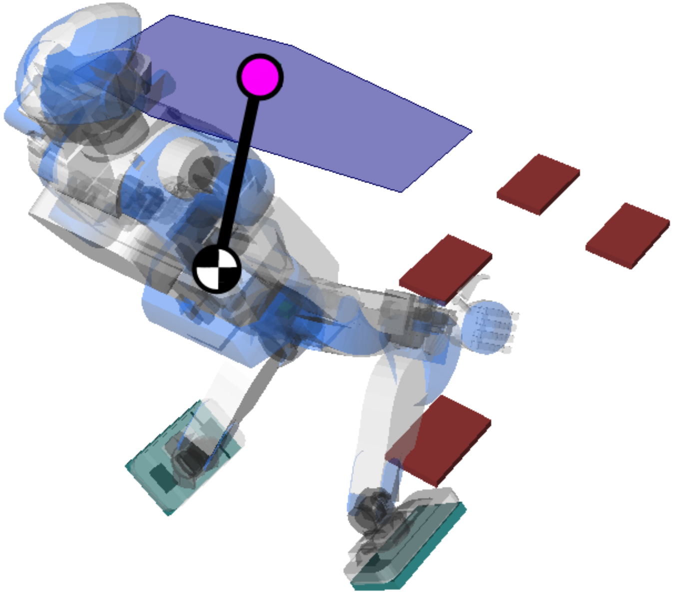

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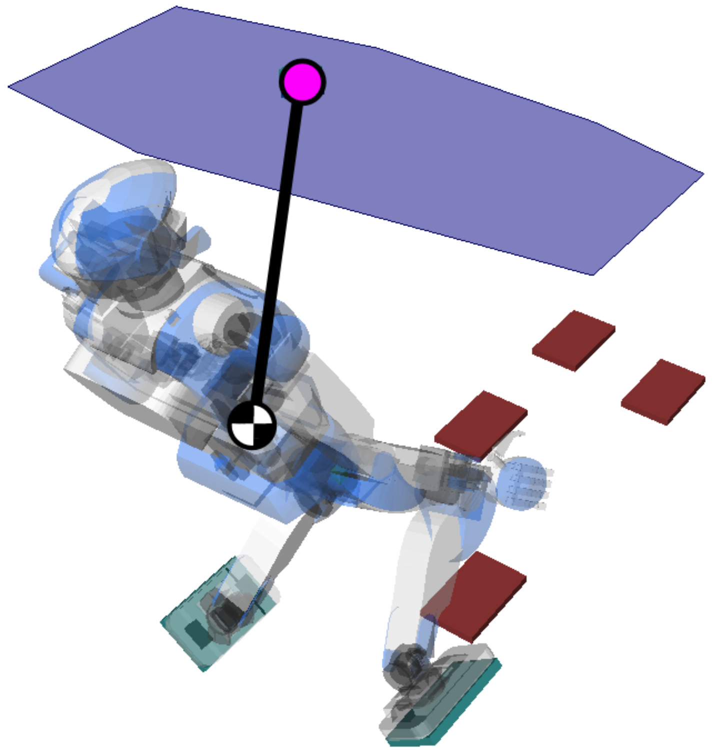
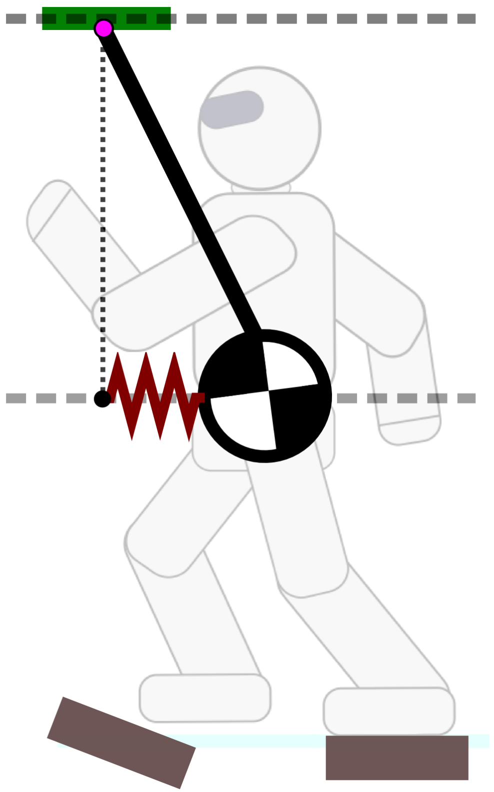


LINEAR PENDULUM MODE

We can now consider the ZMP *above* the COM ⇒ Linear (non-inverted) Pendulum Mode:

$$\ddot{x}_G = \frac{g}{h}(x_G - x_Z)$$

This is the dynamic equation of a spring.



Attractivity

- LIPM: the ZMP is a **repellor** of the COM
- LPM: the ZMP is a marginal **attractor** of the COM

The robot is driven from above, controlling its target position.

MERCI POUR VOTRE ATTENTION.