Between model-based and visuo-motor motion control

Stéphane Caron

Habilitation à diriger des recherches

18 September 2024

This defense

- Past ideas
- Scientific activity
- Next ideas

Equations of motion

Let us start on familiar ground:

- $q \in \mathcal{C}$: configuration of our robot.
- $v, a \in \mathcal{T}_q \mathcal{C}$: velocity and acceleration, tangent vectors.
- Active contact constraints $\varphi(q) = 0$ with Jacobian J and Hessian H:

$$\begin{aligned} \textbf{Kinematics:} \quad & J(q)a + v^\top H(q)v = 0 \\ \textbf{Dynamics:} \quad & M(q)a + v^\top C(q)v + g(q) = S^\top \tau + J(q)^\top f \end{aligned}$$

- τ: joint torques.
- f: contact forces.

Linear inverted pendulum

Whole-body dynamics:

$$M(q)a + v^\top C(q)v + g(q) = S^\top \tau + J(q)^T f$$

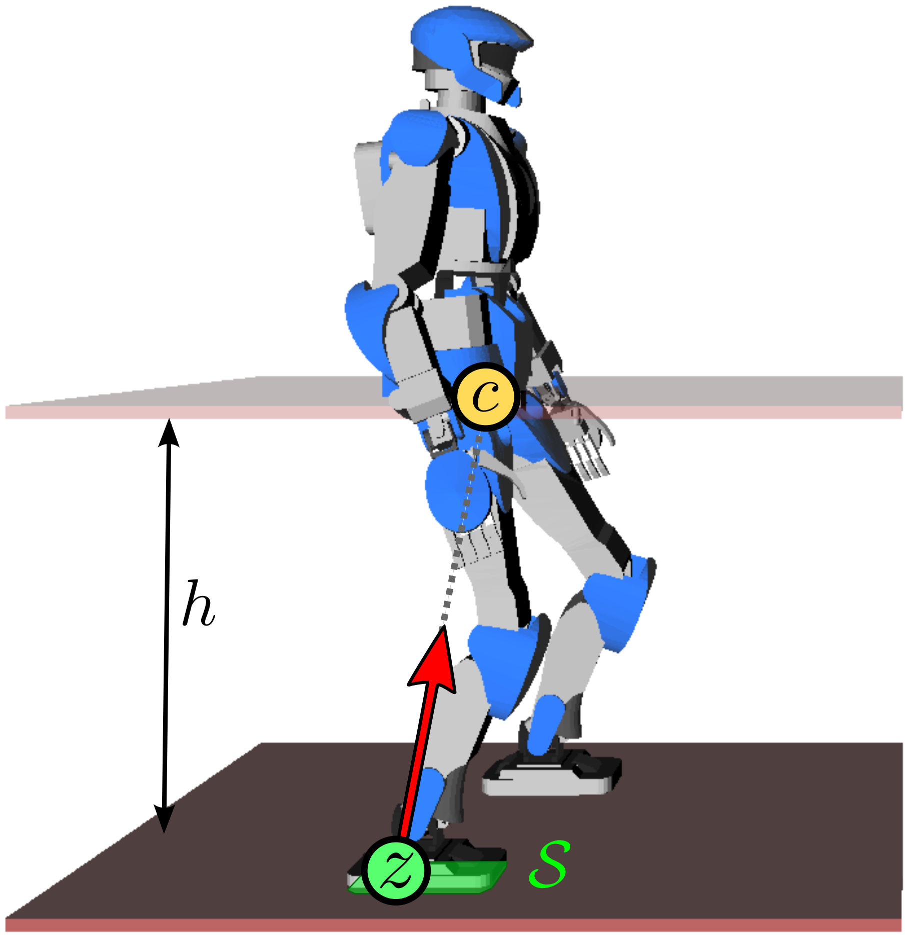
Centroidal dynamics:

$$\ddot{c} = \frac{1}{m} \sum_i f_i$$
$$\dot{L}_c = \sum_i (p_i - c) \times f_i$$

Without angular momentum $\dot{L}_c = 0$:

$$\ddot{c} = \omega^2 (c - z), \quad \omega^2 = \frac{g}{h}$$

where $z \in \mathcal{S}$ is the **zero-tilting moment point**.



Properties of a state

A state $x = [c, \dot{c}]$ of our system is:

Property	Definition	a.k.a.
Feasibility	$\exists z \in \mathcal{S}$ such that the EoM hold	Contact stability
Viability	$\exists z(t)$ such that, integrating the EoM, $\forall t, x(t) \notin \mathcal{F}$ (fall states)	Not falling
Capturability	$\exists z(t)$ such that, integrating the EoM, $\lim_{t \to \infty} \dot{c} = 0$	Plan B = stop

📙 *On the stability of walking systems*, Wieber, 2002 (still a great read in 2024).

Properties of a state so far

A state $x = [c, \dot{c}]$ of our system is:

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📙 *On the stability of walking systems*, Wieber, 2002 (still a great read in 2024).

ZMP support area on flat

The feasibility condition for the ZMP is:

$$z \in \mathrm{conv}(\{p_i\}_i)$$

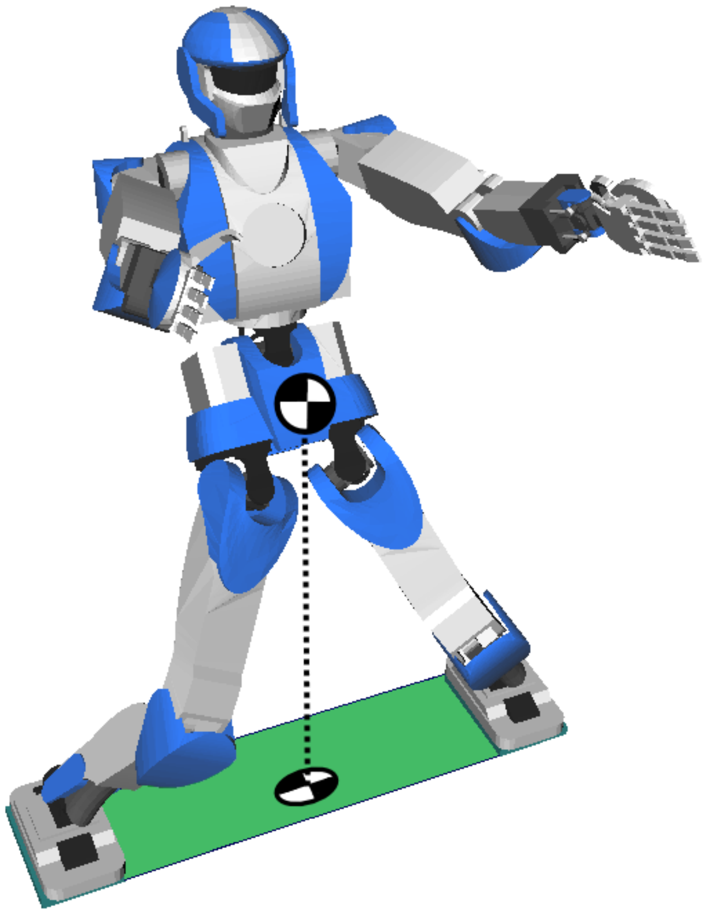
Pros:

- Linear constraint suitable to convex MPC, *e.g.*,
  📙 *Walking without thinking about it*, Herdt *et al.*, 2010.

Cons:

- Assumes coplanar contact points
- Assumes infinite friction

Support areas in general? Computability?



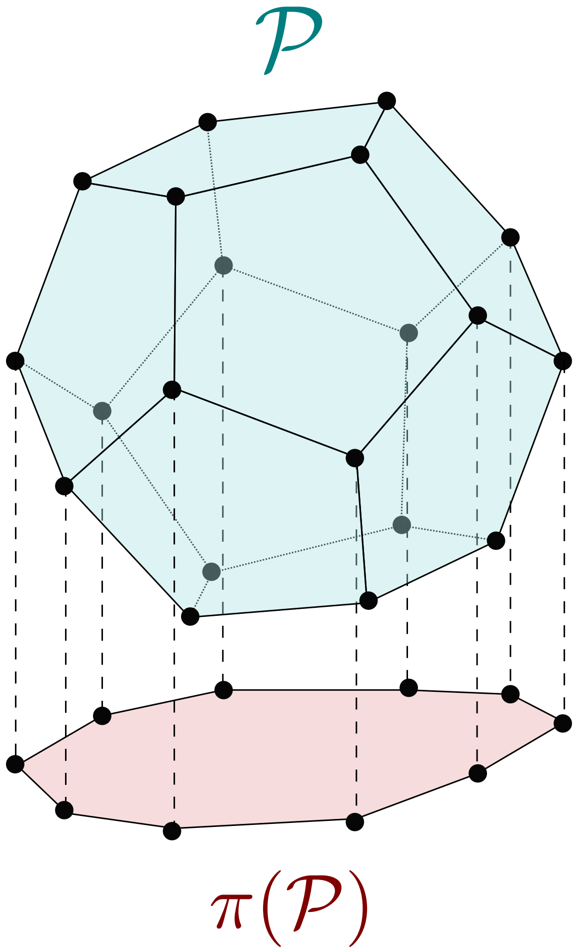
Polytope projection

Polyhedron $\mathcal{P} = \{x \in \mathbb{R}^m, y \in \mathbb{R}^n, Ax + By \leq c\}$

Projection $\pi(\mathcal{P})$: $x \in \pi(\mathcal{P}) \Leftrightarrow \exists y, (x, y) \in \mathcal{P}$

Some algorithms:

- Fourier-Motzkin elimination (Fourier, 1827), $O(h^{2^k})$
- Double description (Fukuda & Prodon, 1996), $O(h^2 v^3)$
- Convex hull, *e.g.* by pivoting (Avis, Fukuda, 1992), $O(dhv)$
- Output-sensitive projection algorithms

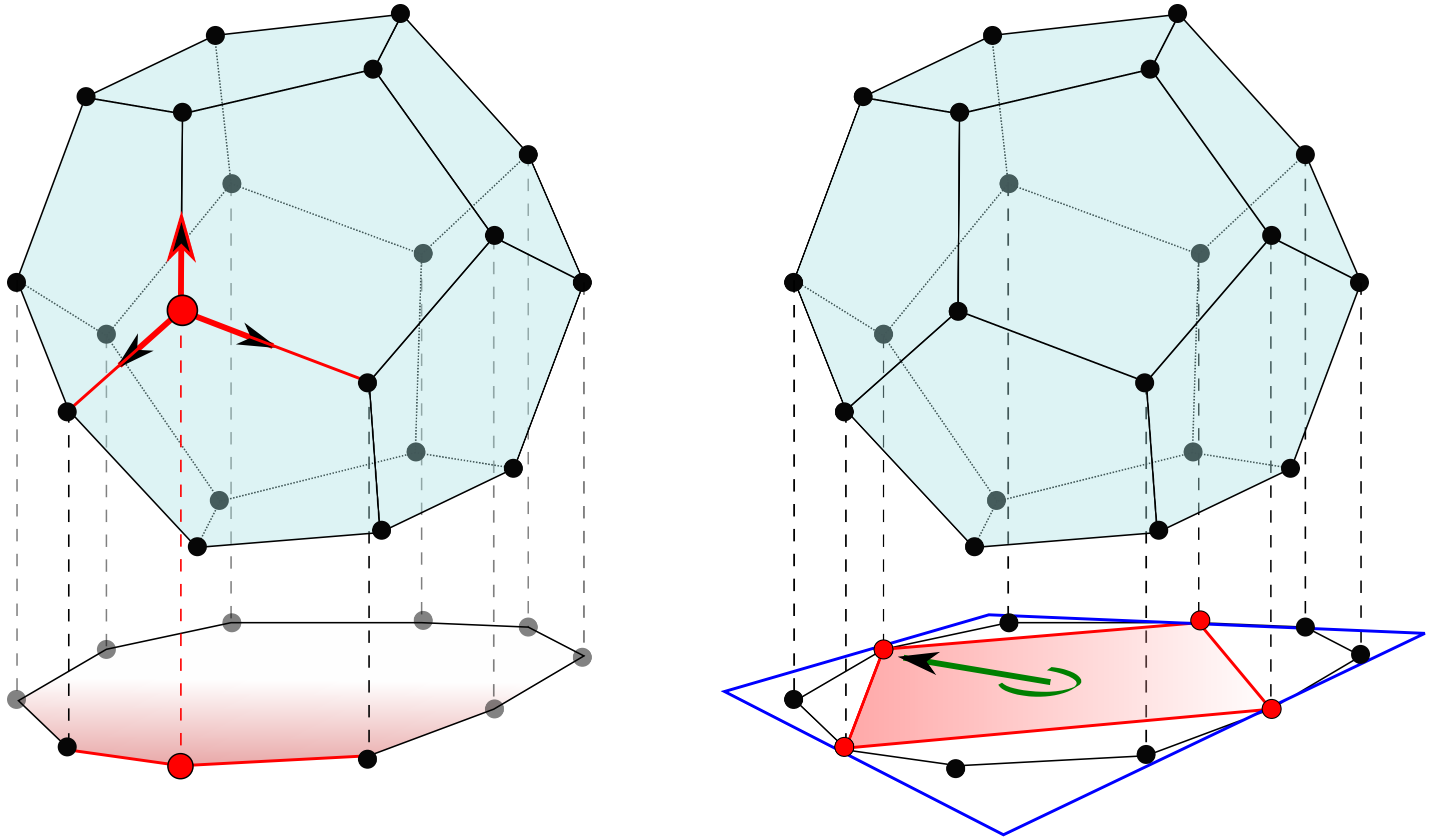


Output-sensitive algorithms

Complexity depends on size of *projected* polytope

Algorithms:

- Contour tracking (Ponce *et al.*, 1997)
- Outer-inner ray casting (Bretl & Lall, 2006, pypoman)



Contour tracking　　　　Ray casting

ZMP from contact forces

The ZMP can be expressed from contact forces f as:

$$z = Gf + z_0$$

Contact forces are subject to Coulomb friction:

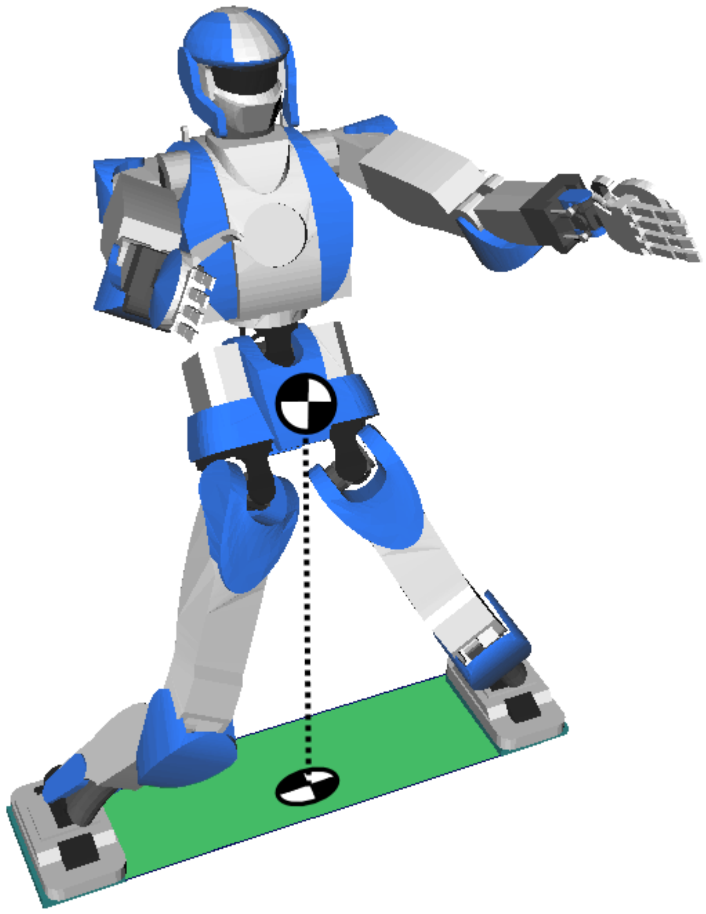
$$Af \leq 0$$

Polytope projection yields a ZMP support area :

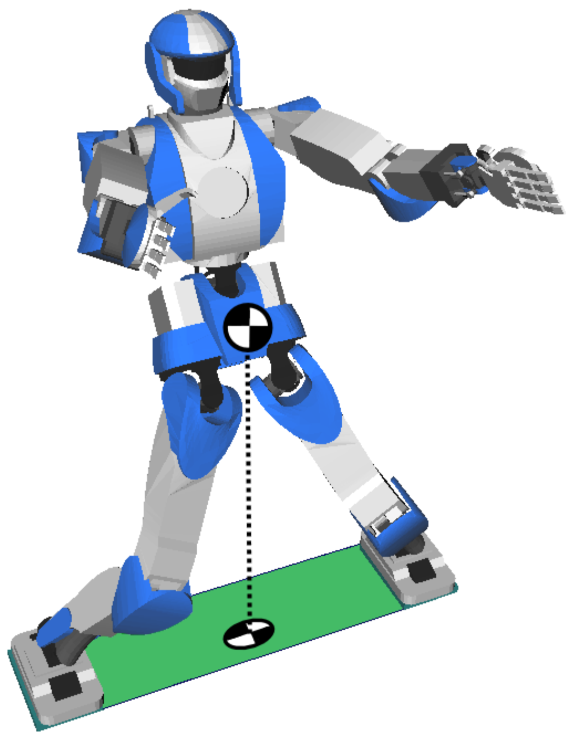
$$\exists f, \begin{bmatrix} A & 0 \\ \mp G & \pm I \end{bmatrix} \begin{bmatrix} f \\ z \end{bmatrix} \leq \begin{bmatrix} 0 \\ \pm z_0 \end{bmatrix} \Longleftrightarrow Hz \leq b$$

No assumption of coplanar contacts or infinite friction.

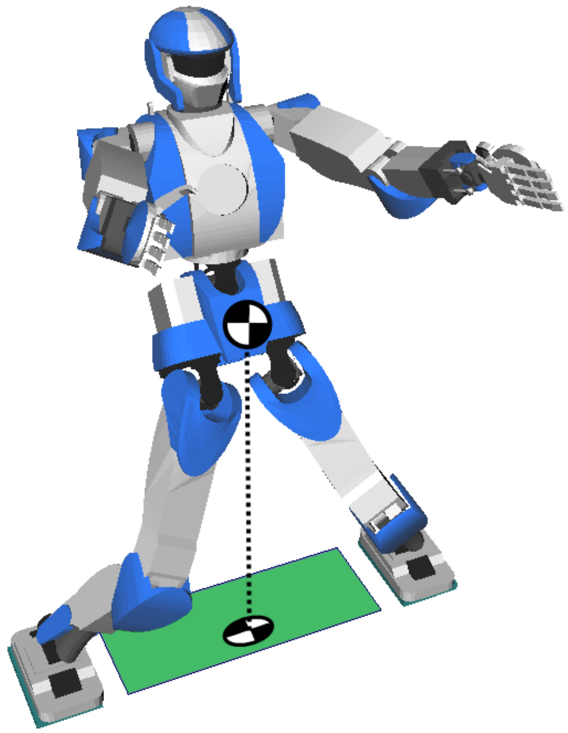
📙 *ZMP Support Areas for Multicontact Mobility Under Frictional Constraints*, Caron, Pham & Nakamura, 2016.



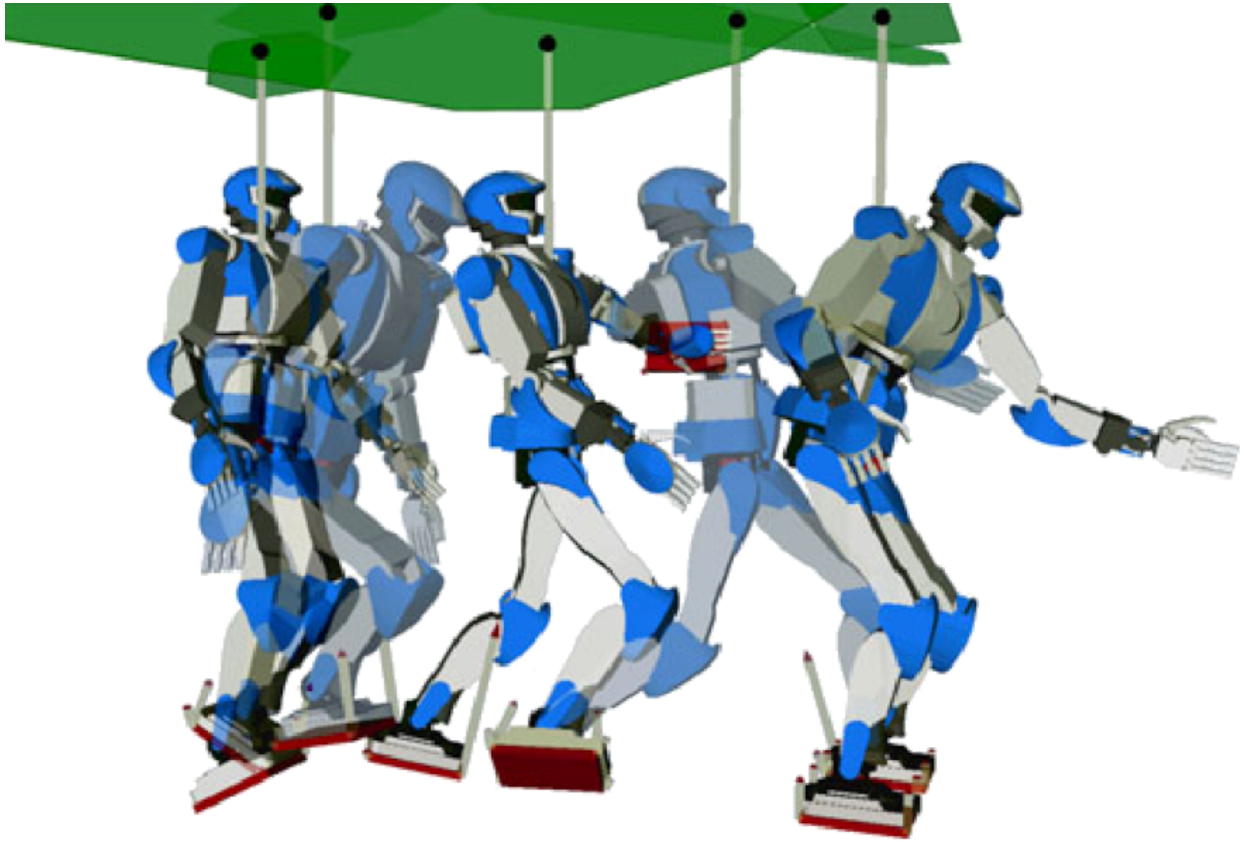
ZMP support areas



ZMP support areas



Multi-contact ZMP support areas



Support volumes

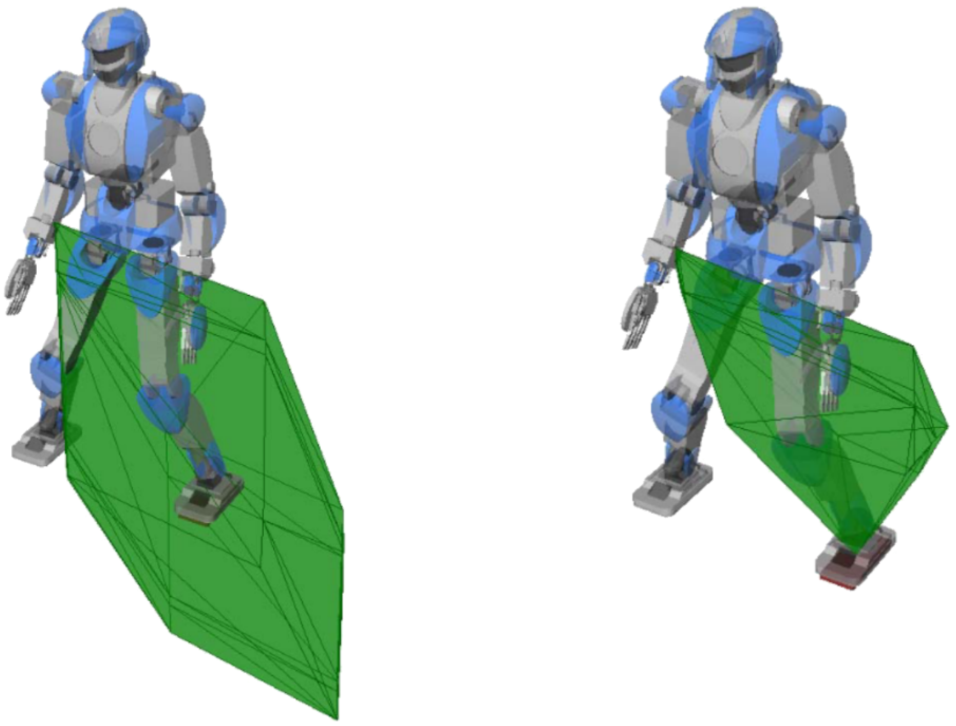
I have continued working on this topic :

- Double-description for net wrench friction cones (Caron, Pham & Nakamura, RSS 2015)
- Reduce net wrench cones to centroidal ones (Caron & Kheddar, Humanoids 2016)
- Explicit reduced-model cone in single-support phases (Caron & Kheddar, IROS 2017)
- Torque-limited friction cones and ZMP areas (Orsolino *et al.*, T-RO 2020)

Shout out to related works:

- *3-D robust stability polyhedron in multicontact* (Audren & Kheddar, 2017)
- Torque limits projected as workspace polytopes (Skuric, Padois & Daney, 2022, PyCapacity)
- *CROC: Convex Resolution Of Centroidal dynamics trajectories* (Fernbach, Tonneau & Taïx, 2018) 🤩

Torque-limited friction polytopes



Left: leg torque limits. **Right:** leg-torque and friction limits.

Properties of a state so far

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Properties of a state thereafter

A state $x = [c, \dot{c}]$ of our system is:

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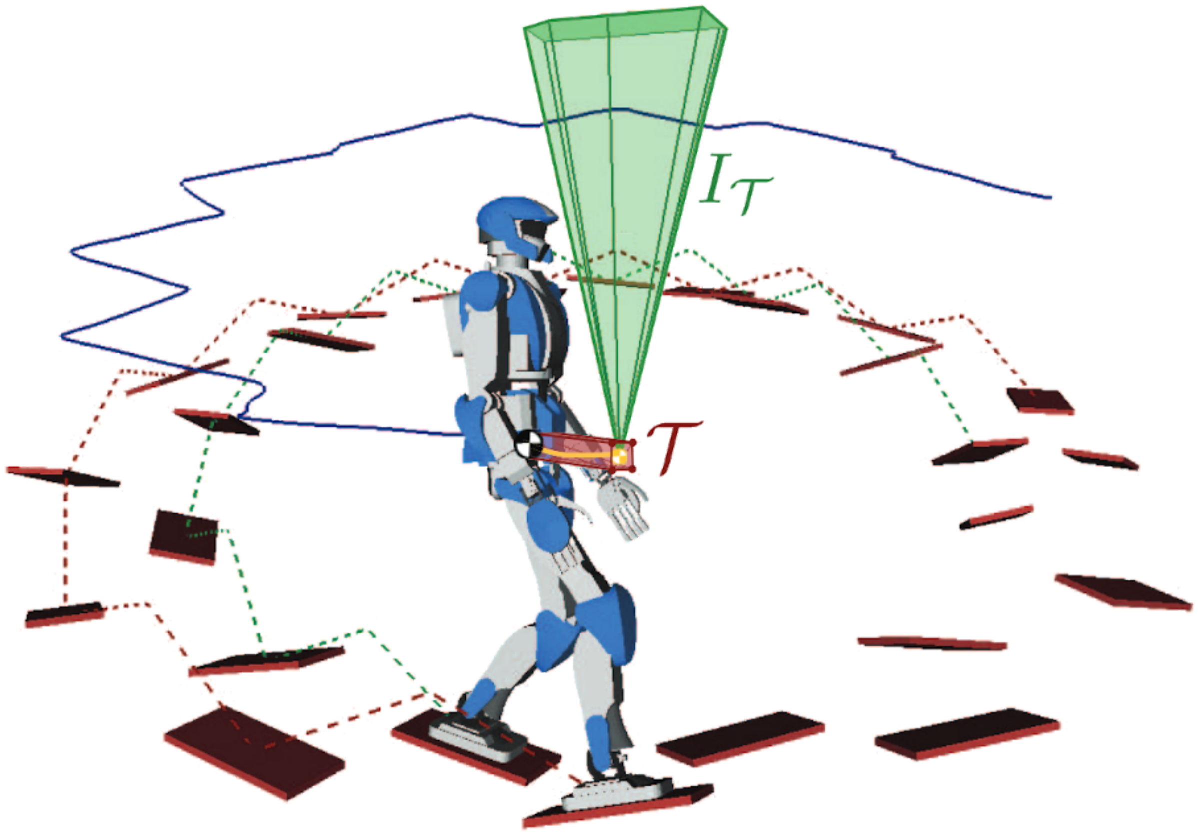
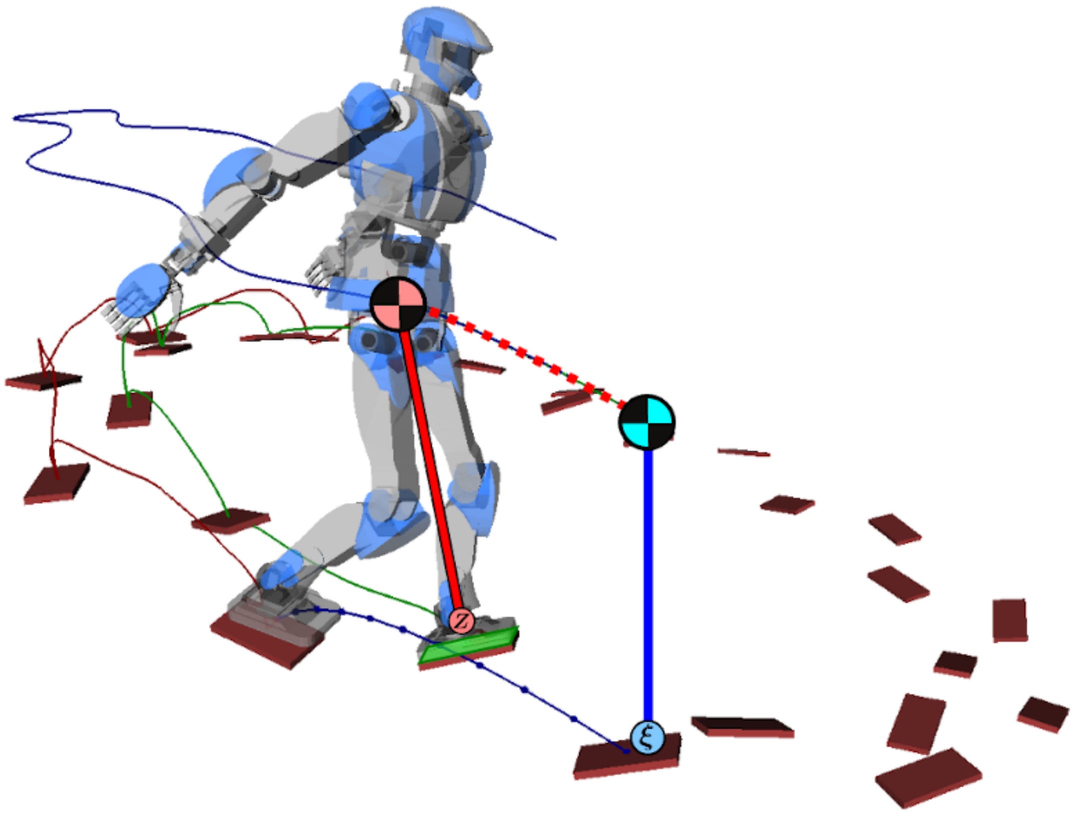
Capturability for robot locomotion

Conservative constraint linearization



Multi-contact walking pattern generation based on ... 3D COM accelerations, Caron & Kheddar, 2016.

Nonlinear model predictive control

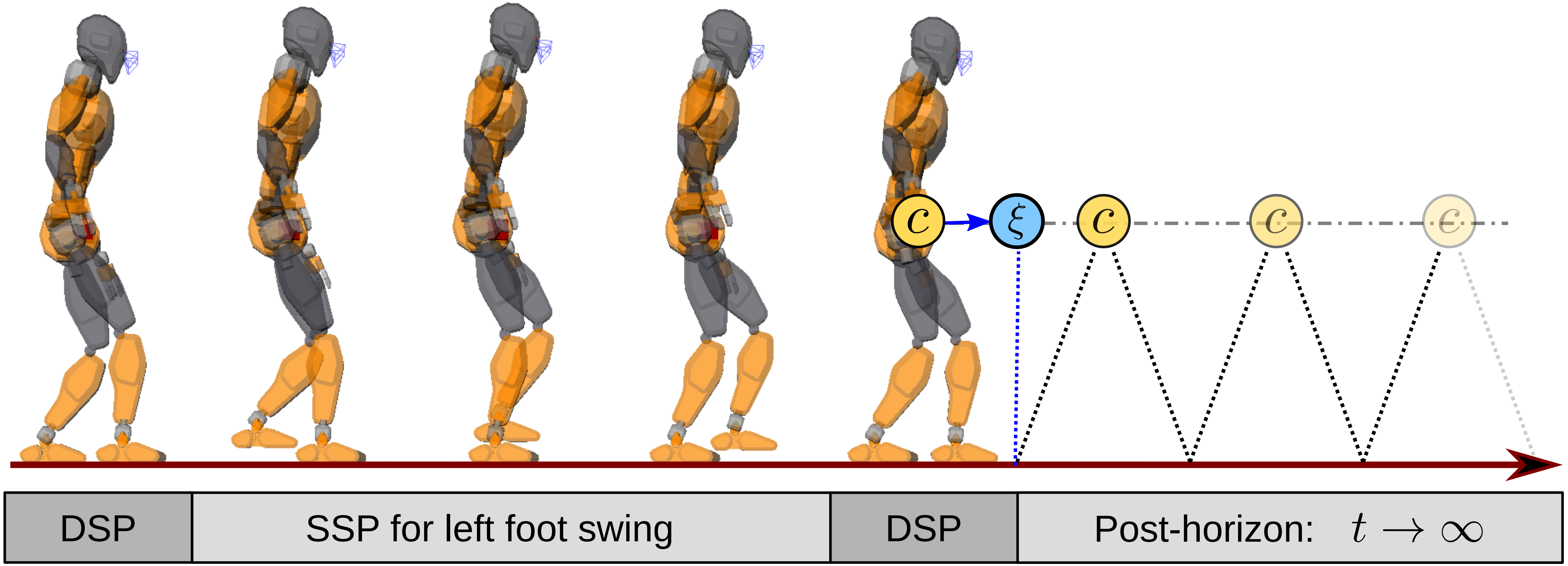


Dynamic Walking over Rough Terrains by Nonlinear Predictive Control ..., Caron & Kheddar, 2017.

How to avoid "running out of plans"?

Recursive feasibility: if MPC problem is feasible at iteration t, then it is feasible at iteration $t + 1$.

- Typically enforced via a *terminal constraint* (Mayne, 2014)
- Walking: capturability constraint yields recursive feasibility (Ciocca, Wieber, Fraichard, 2017)
- More general than capturability: *boundedness constraint* (Lanari, Hutchinson, Marchionni, 2014)


c
ξ
c
c
c
DSP
SSP for left foot swing
DSP
Post-horizon: $t \to \infty$

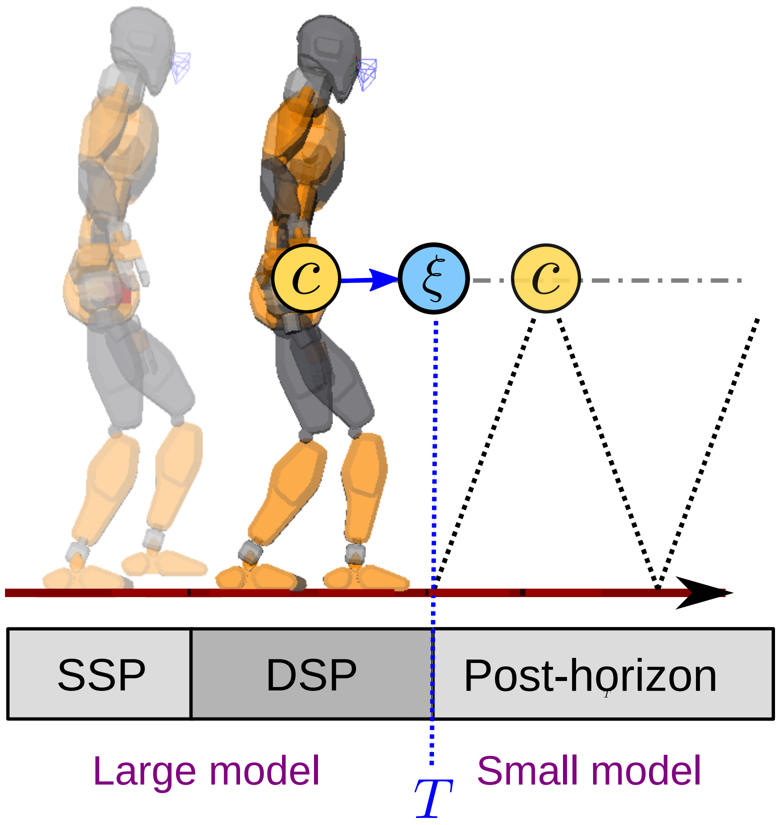
Boundedness condition

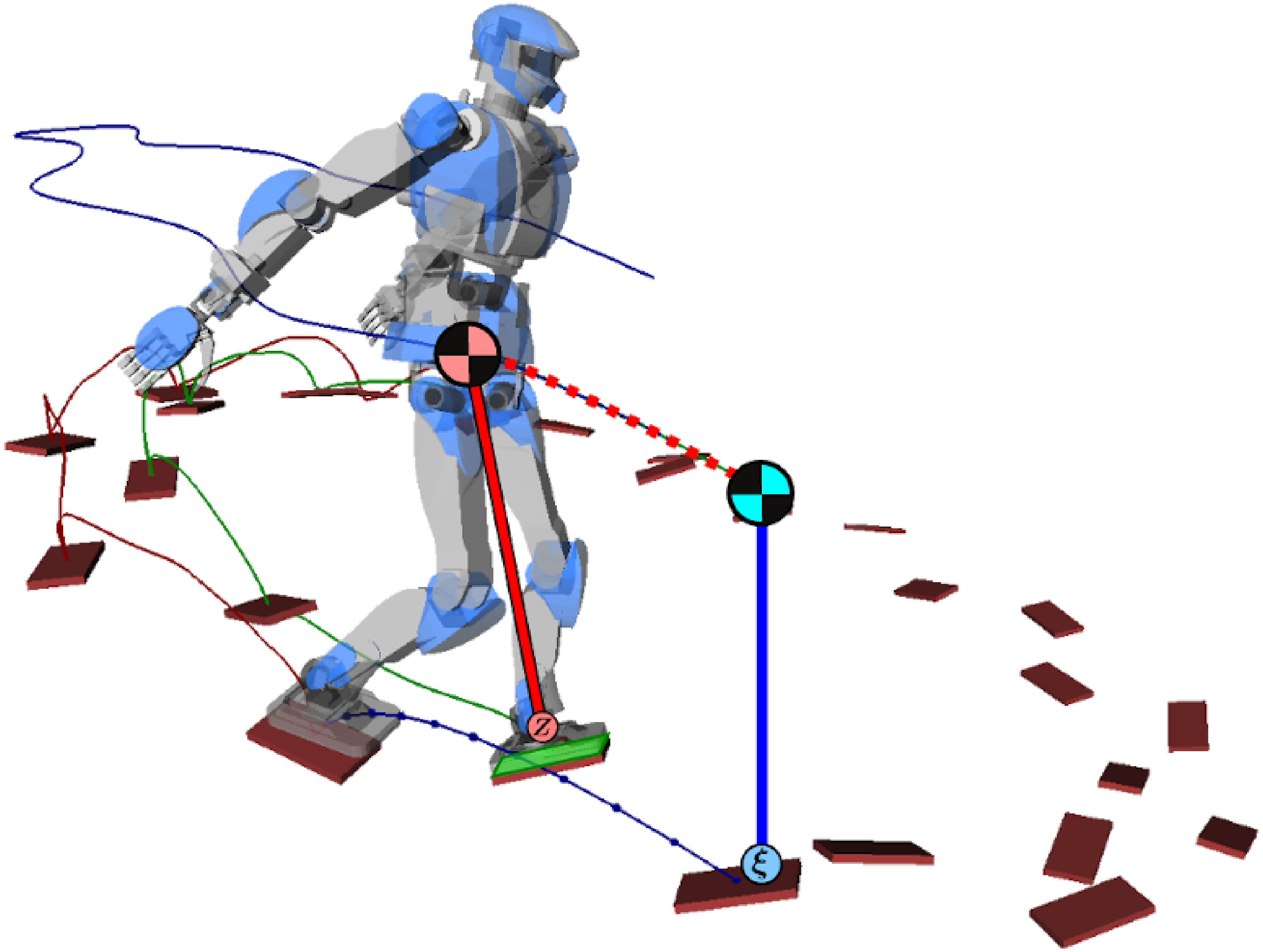
- Generalizes $\lim_{t \to \infty} \dot{c} = 0$ to post-horizon inputs
- Switch to model simple enough to solve analytically
- For the linear inverted pendulum model:

$$\xi(x_T) = \omega \int_{t=T}^{\infty} e^{-\omega t} z(t) \mathrm{d}t$$

- Terminal constraint for the MPC problem

📙 *Boundedness issues in planning of locomotion trajectories for biped robots*, Lanari, Hutchinson, Marchionni, 2014.





Variable-height inverted pendulum

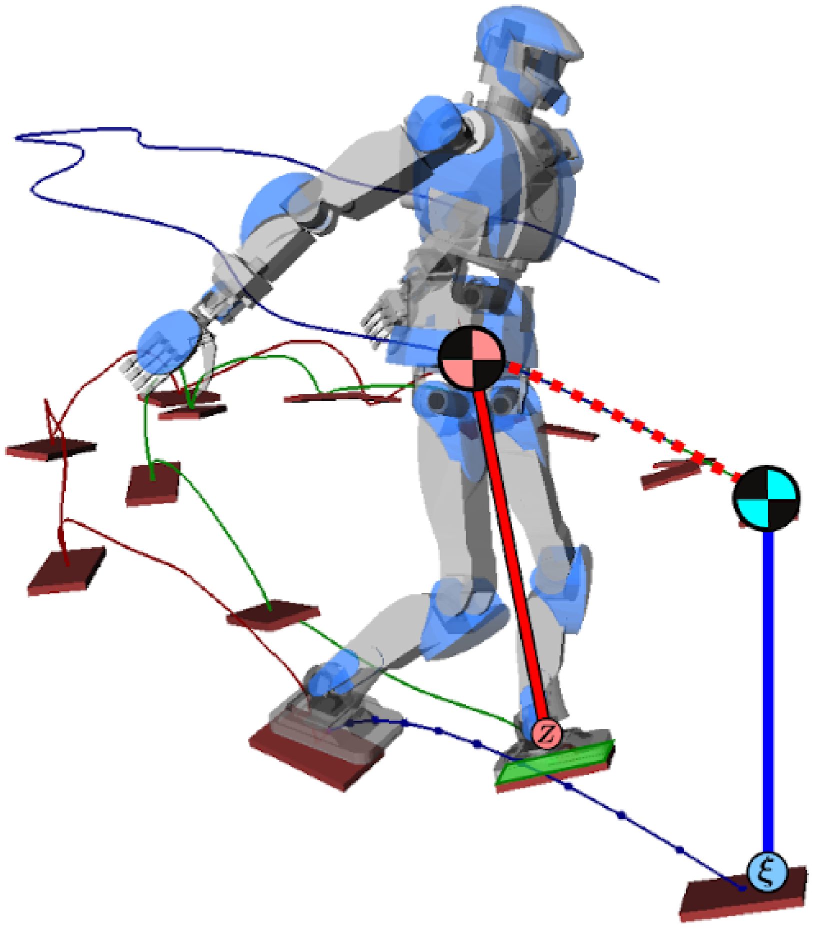
- Fixed-height: $\ddot{c} = \omega^2 (c - z) + g$
- Variable-height: $\ddot{c} = \lambda (c - z) + g$ (Koolen *et al.*, 2016)
- Boundedness condition:

$$\xi(x_T) = \int_{t=T}^{\infty} e^{-\Omega(t)} (\lambda(t) r(t) - g) \mathrm{d}t$$

- where we introduced $\dot{\omega} = \lambda - \omega^2$ (Hauser *et al.*, 2004)
- Change variable to $s(t) = e^{-\Omega(t)}$:

$$\xi(x_T) = \int_{s=0}^{1} \left[r(s)(s\omega)' - g\omega(s)^{-1} \right] \mathrm{d}s$$

📙 *Capturability-Based Pattern Generation for Walking With Variable Height*, Caron, Escande, **Lanari**, **Mallein**, 2019.



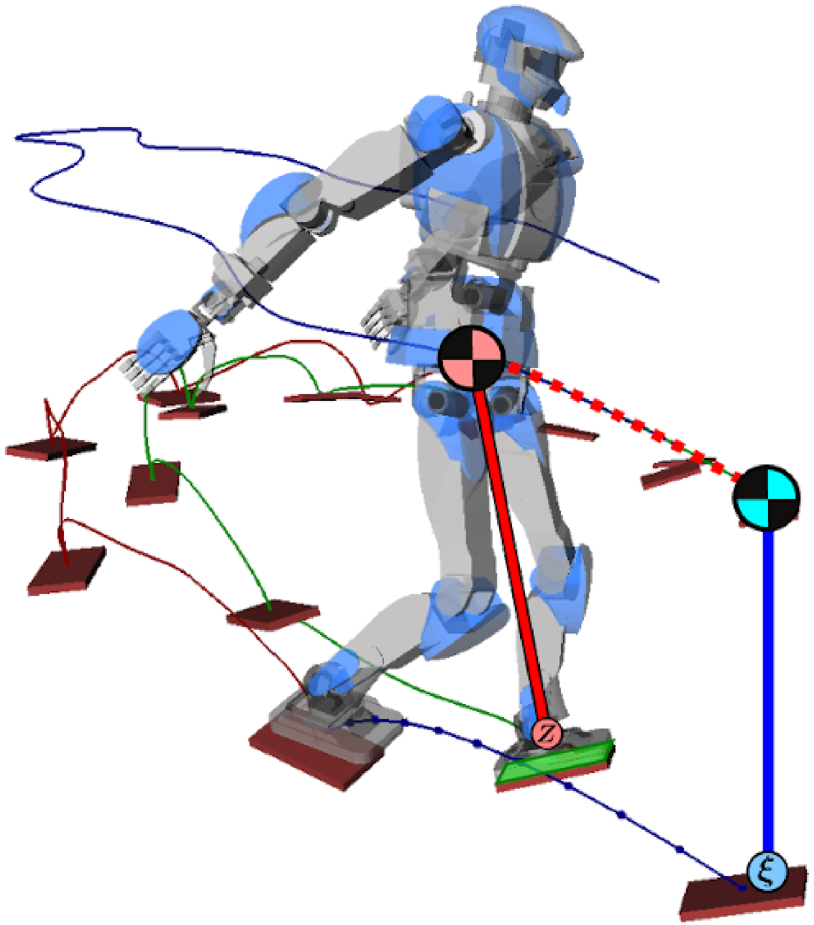
Capturable VHIP trajectories

Massage model predictive control problem into:

$$\begin{aligned} \underset{\varphi \in \mathbb{R}^n}{\text{minimize}} \quad & \frac{1}{2} \varphi^T P \varphi + q^T \varphi \\ \text{subject to} \quad & A\varphi \leq b \\ & \sum_{j=0}^{n-1} \frac{\delta_j}{\sqrt{\varphi_{j+1}} + \sqrt{\varphi_j}} - \frac{h_T \sqrt{\varphi_n} + \dot{h}_T}{g} = 0 \end{aligned}$$

- *Almost* a convex quadratic program (QP)
- Ludicrously fast dedicated SQP solver (Escande, CaptureProblemSolver): ∞-horizon in ~0.03 ms

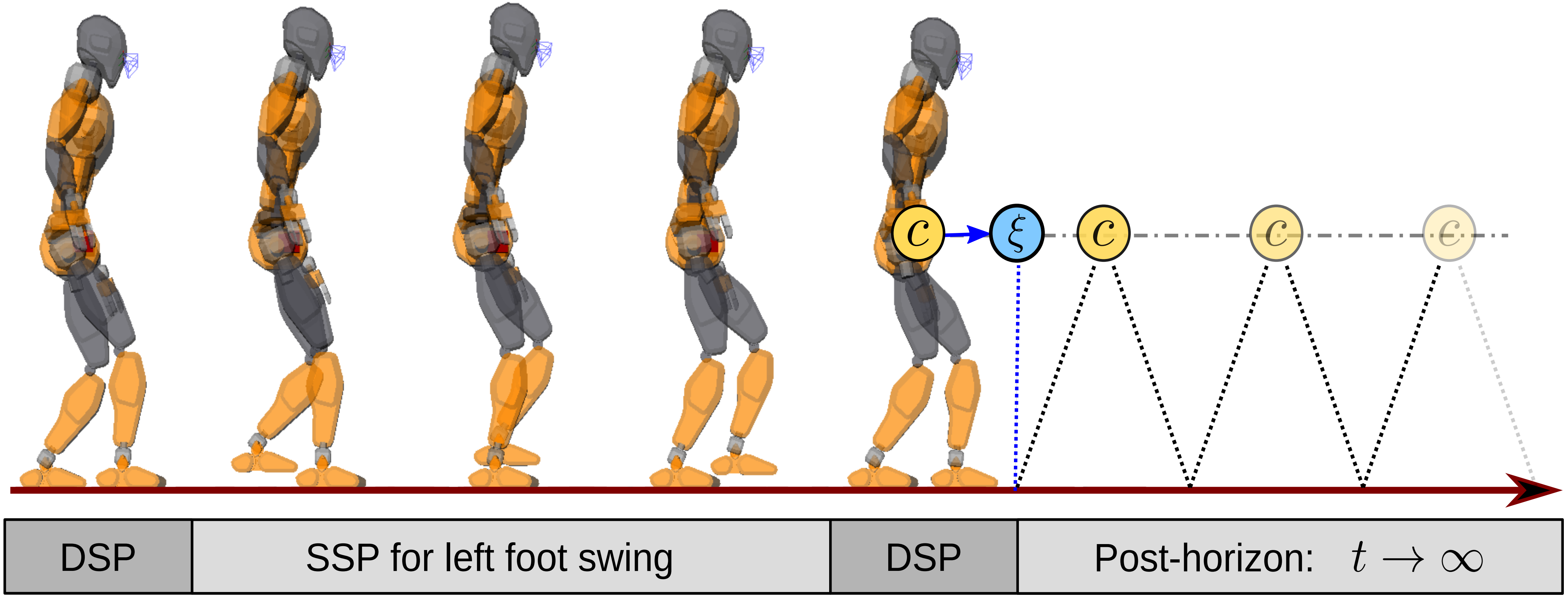
📙 *Capturability-Based Pattern Generation for Walking With Variable Height*, **Caron**, **Escande**, Lanari, Mallein, 2019.



A. Escande

L. Lanari

B. Mallein



c
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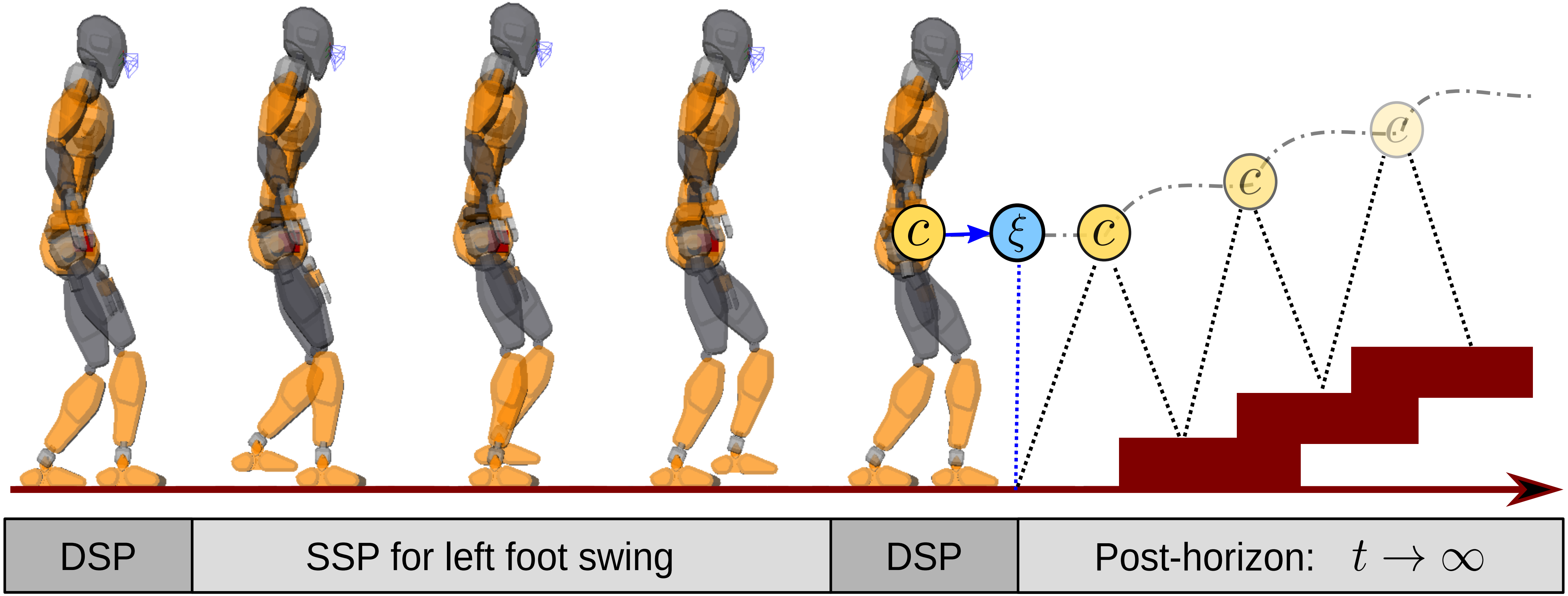
SSP for left foot swing
DSP
Post-horizon: $t \to \infty$


c
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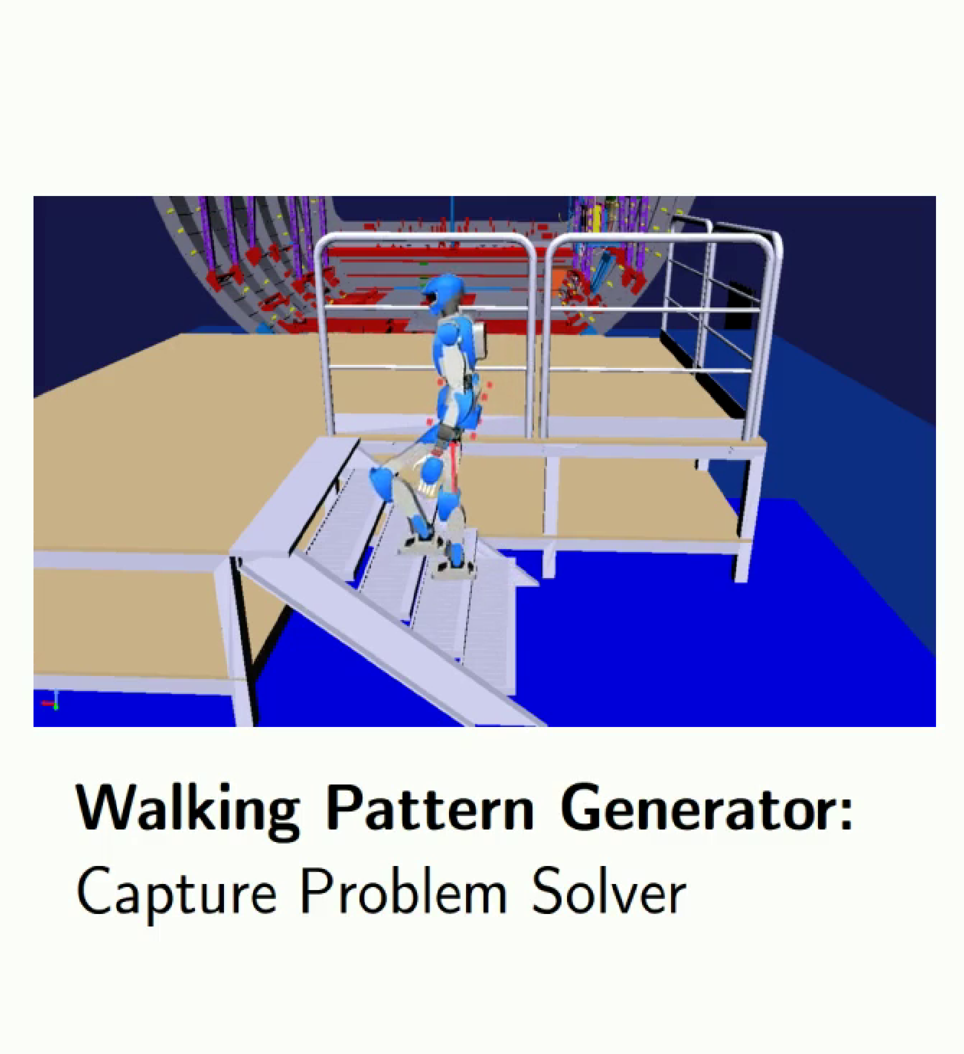
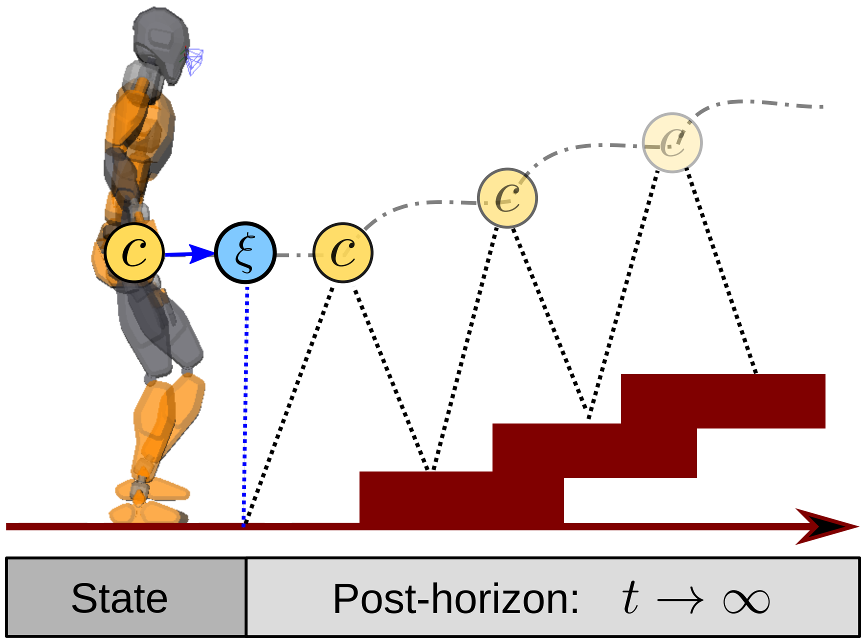
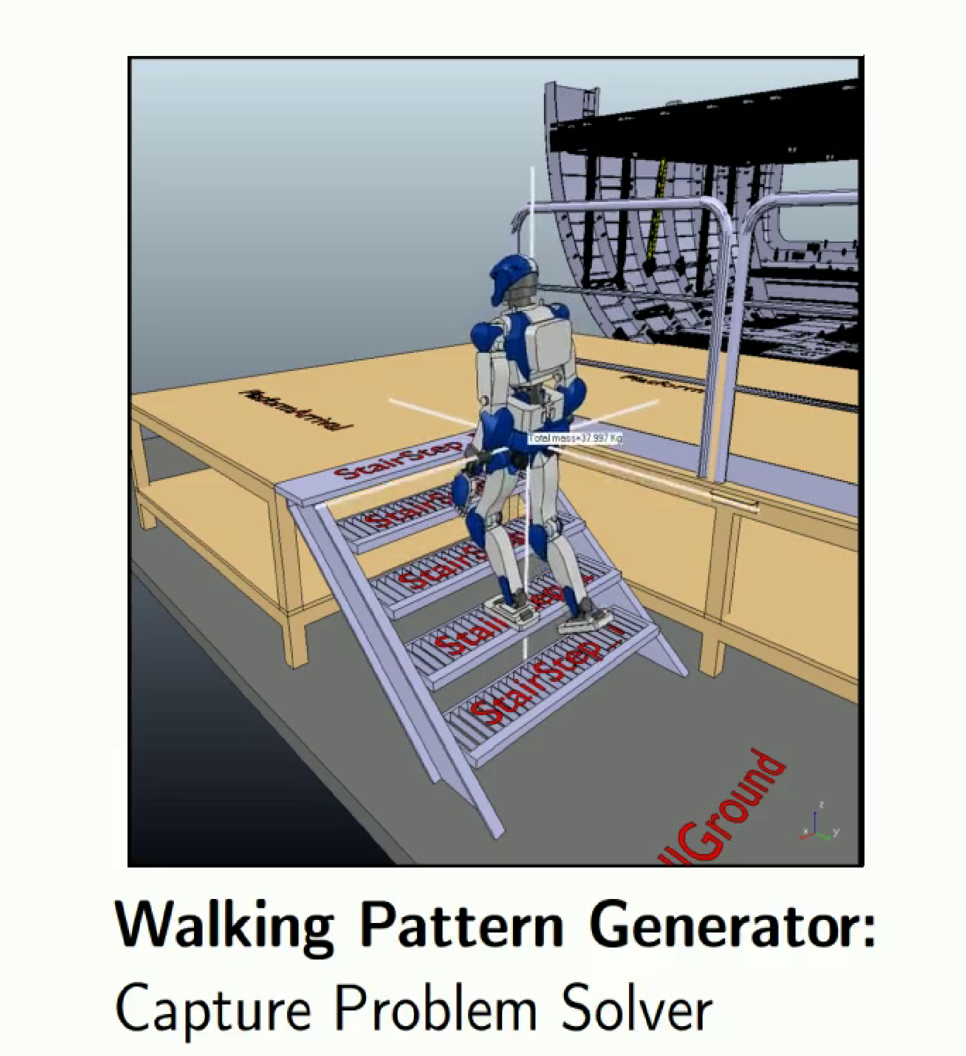
DSP
Post-horizon: $t \to \infty$

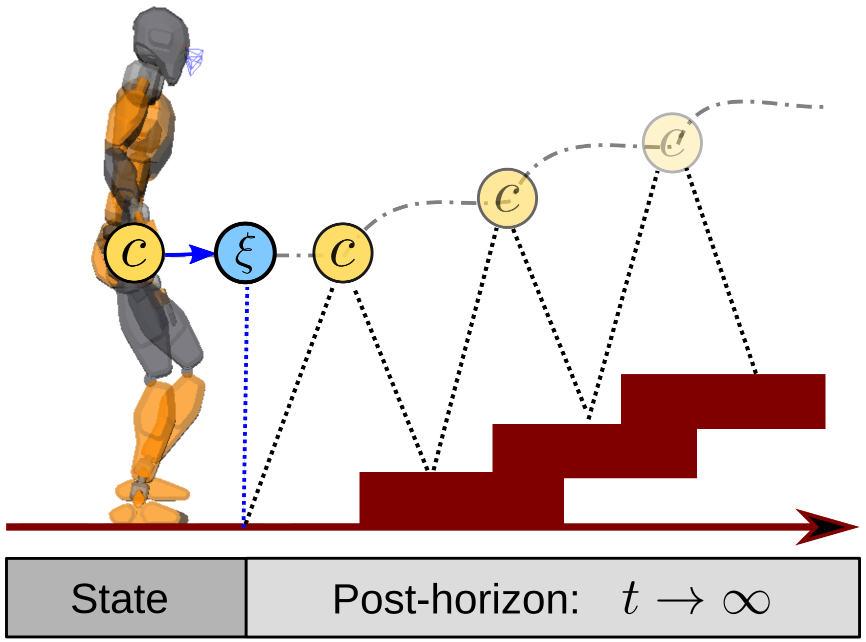
Post-horizon-only MPC





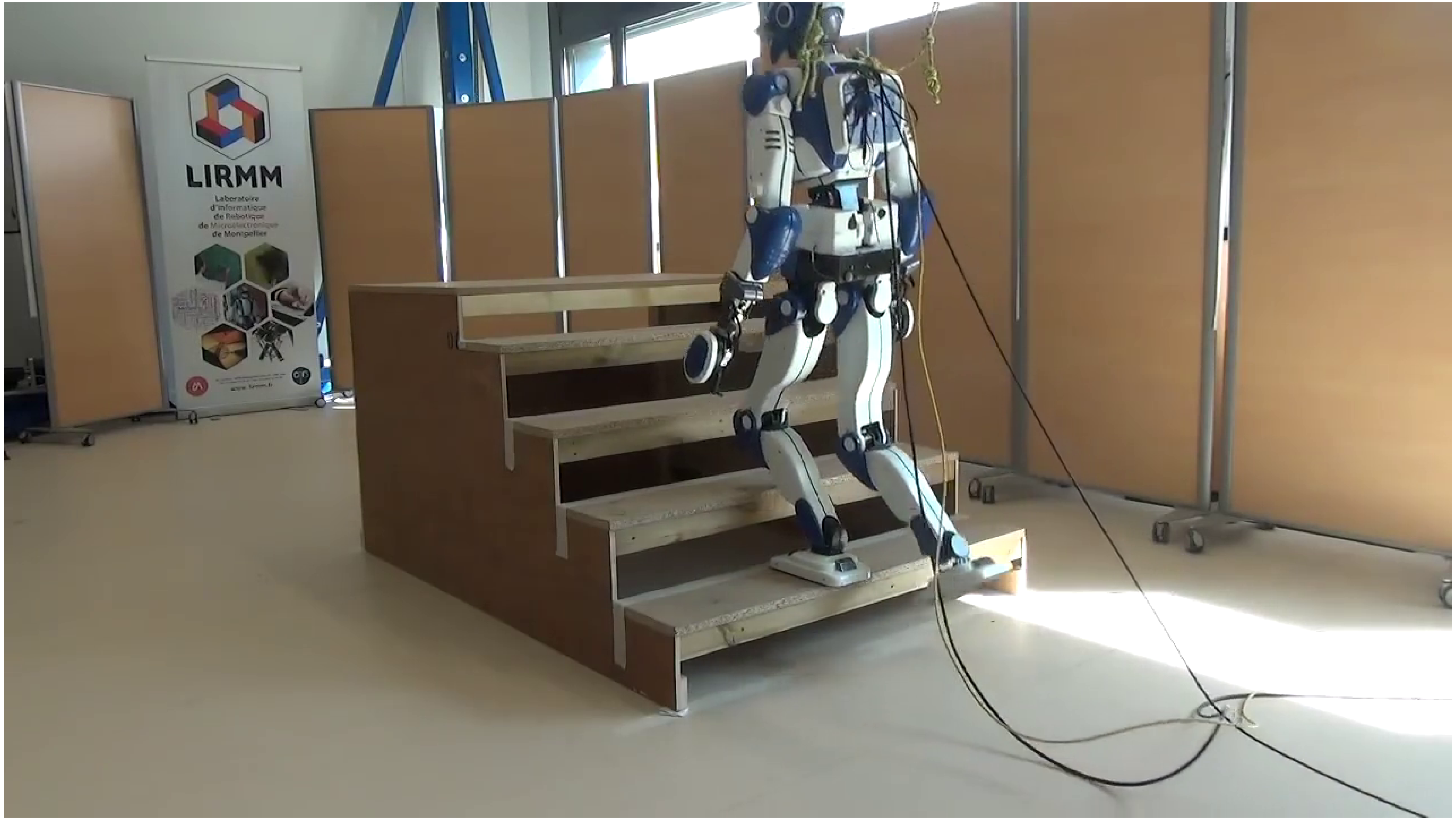
Post-horizon-only MPC

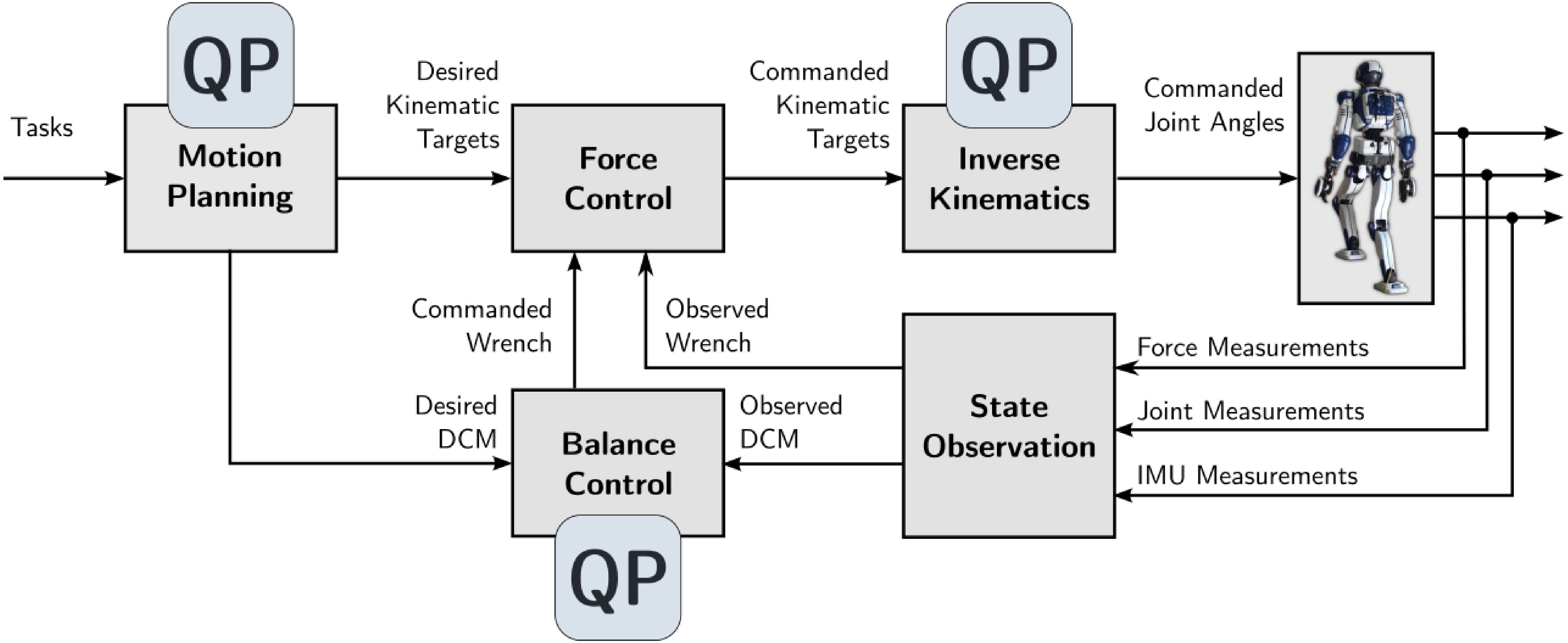




In a parallel thread

Stair climbing

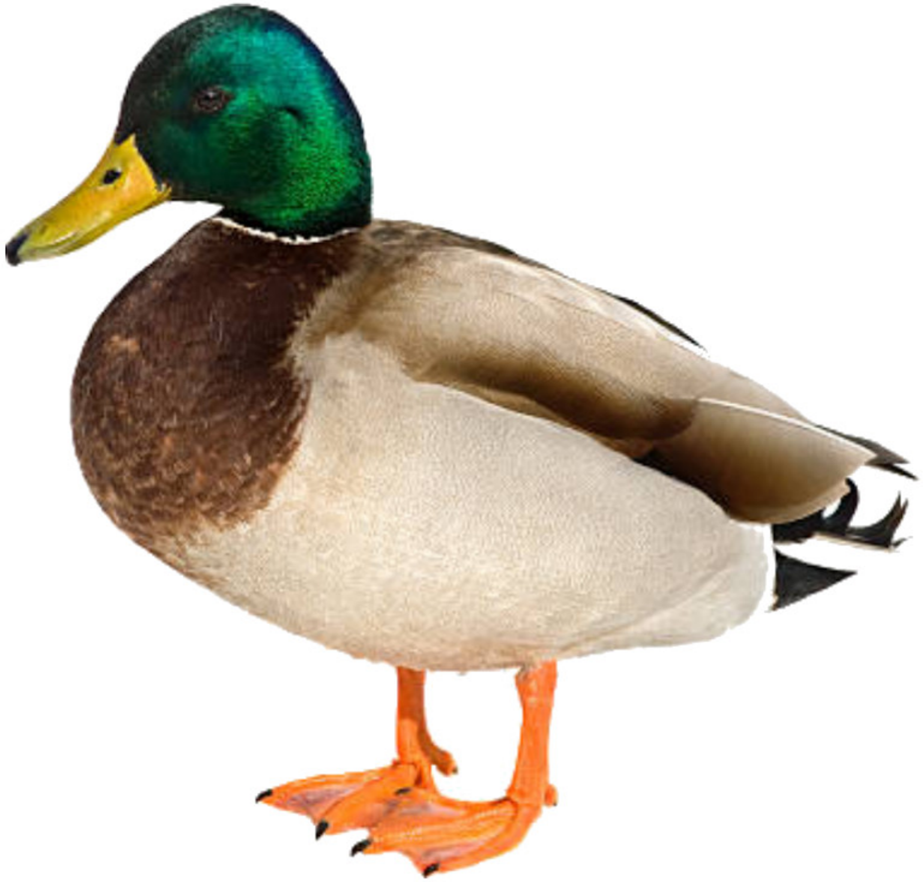



Tasks
QP
Motion Planning
Desired Kinematic Targets
Force Control
Commanded Kinematic Targets
QP
Inverse Kinematics
Commanded Joint Angles
Commanded Wrench
Observed Wrench
Force Measurements
Desired DCM
Balance Control
QP
Observed DCM
State Observation
Joint Measurements
IMU Measurements

LIPM walking controller

- Successor to LIP tracking (Kajita *et al.*, 2010)
- Reformulate heuristics as constrained optimization
- Open source: lipm_walking_controller
  - P. Gergondet (mc_rtc)
  - J. Vaillant (Tasks)
  - *et al.*: K. Chappellet, A. Tanguy, ...

📙 *Stair Climbing Stabilization of the HRP-4 Humanoid Robot using Whole-body Admittance Control*, Caron, Kheddar & Tempier, 2019.



(click on the duck to follow the trail)

ANYbotics AG

ANYbotics AG. (2021, Apr 21). ANYbotics Introduces End-to-End Robotic Inspection Solution. YouTube.

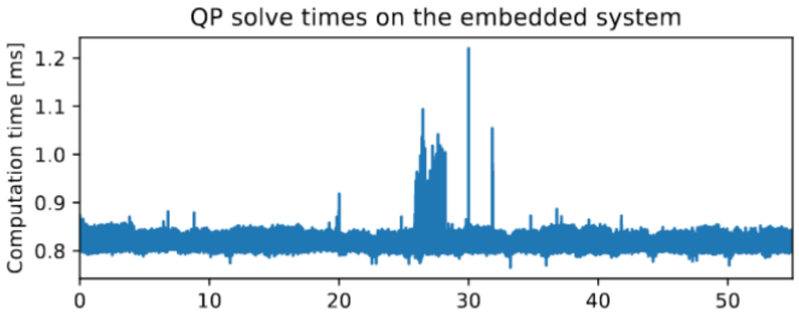
ProxSuite

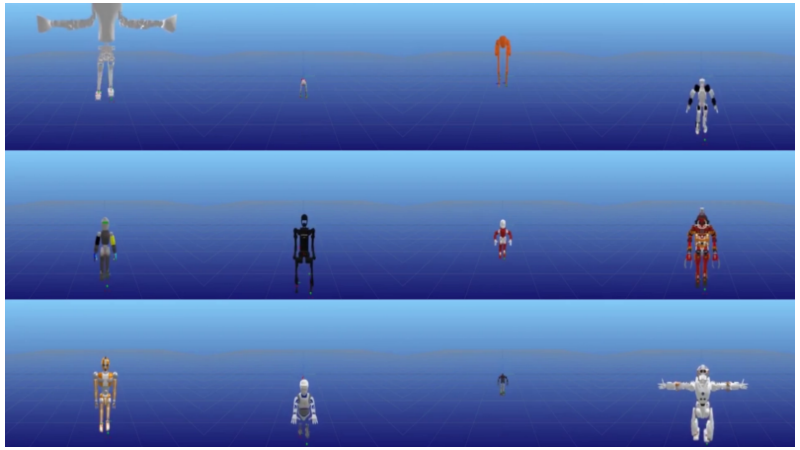
THE ADVANCED PROXIMAL OPTIMIZATION TOOLBOX

Willow team at Inria

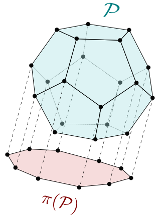
Joined Willow in January 2023 after a three-year hiatus:

- *PROXQP: an Efficient and Versatile Quadratic Programming Solver for Real-Time Robotics Applications and Beyond*, Bambade, *et al.*, 2023.
- *Linear-time Differential Inverse Kinematics: an Augmented Lagrangian Perspective*, Wingo *et al.*, RSS 2024.

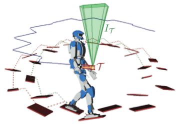




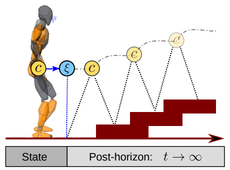
Contact
wrench cones



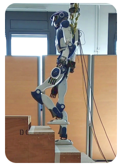
Multi-contact
ZMP support areas



Conservative
constraint
linearization

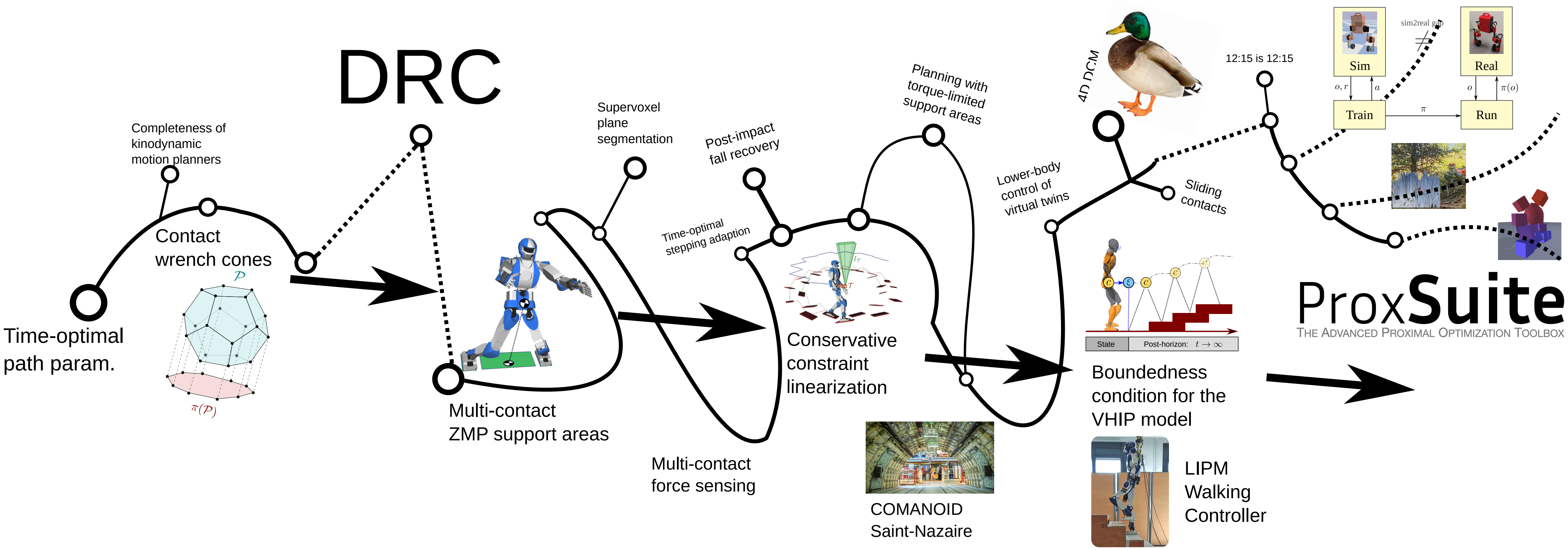


Boundedness
condition for the
VHIP model



LIPM
Walking
Controller

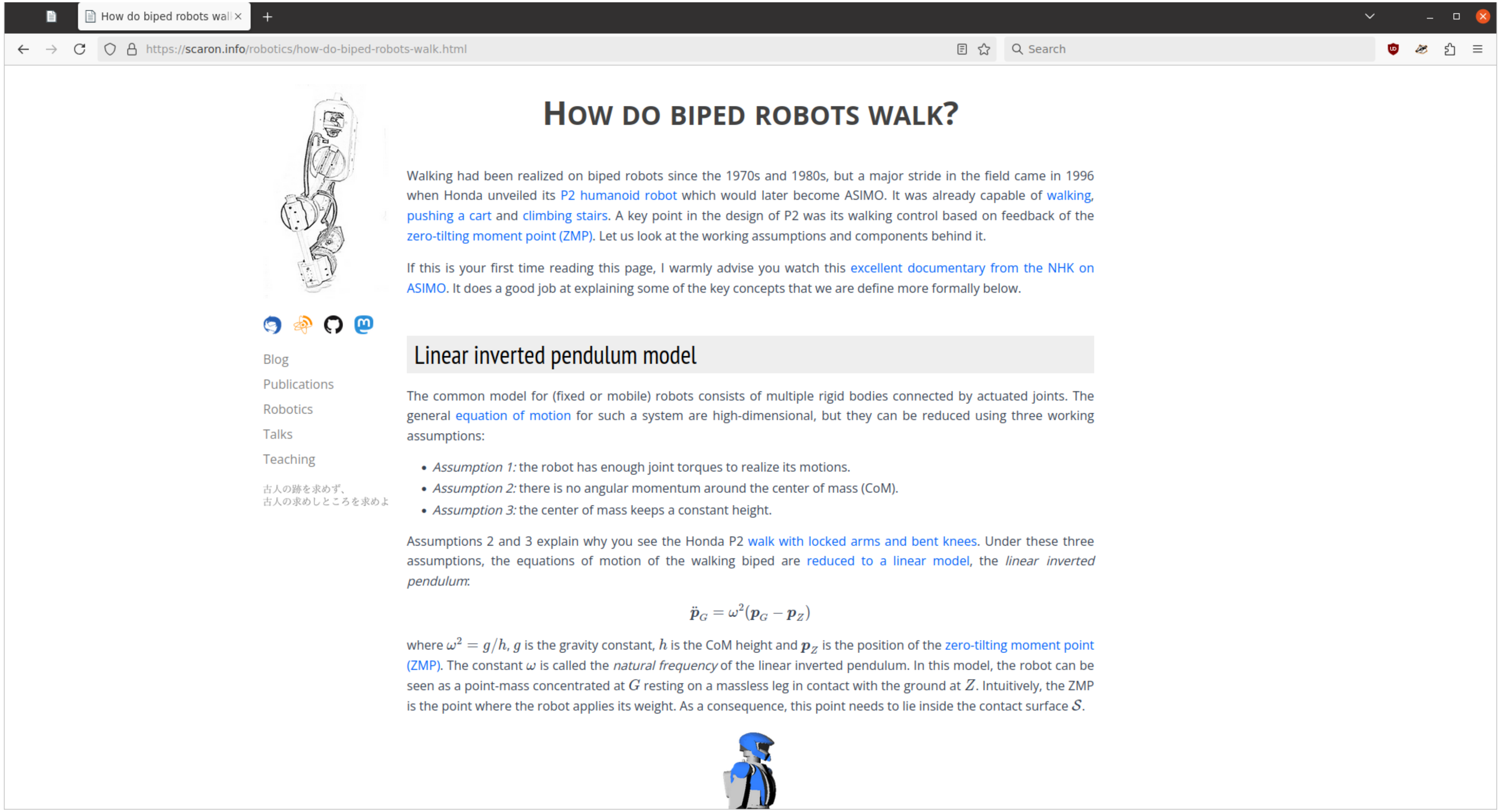
ProxSuite
The Advanced Proximal Optimization Toolbox


DRC
Time-optimal path param.
Completeness of kinodynamic motion planners
Contact wrench cones
Multi-contact ZMP support areas
Supervoxel plane segmentation
Post-impact fall recovery
Time-optimal stepping adaption
Multi-contact force sensing
Planning with torque-limited support areas
Conservative constraint linearization
COMANOID Saint-Nazaire
Lower-body control of virtual twins
4D DCM
Sliding contacts
Boundedness condition for the VHIP model
State
Post-horizon:
LIPM Walking Controller
12:15 is 12:15
Sim
Real
Train
Run
sim2real gap
ProxSuite
The Advanced Proximal Optimization Toolbox

Scientific activity

Editorial activities

Website



How do biped robots wal ×

https://scaron.info/robotics/how-do-biped-robots-walk.html

Blog
Publications
Robotics
Talks
Teaching

古人の跡を求めず、
古人の求めしところを求めよ

How do biped robots walk?

Walking had been realized on biped robots since the 1970s and 1980s, but a major stride in the field came in 1996 when Honda unveiled its P2 humanoid robot which would later become ASIMO. It was already capable of walking, pushing a cart and climbing stairs. A key point in the design of P2 was its walking control based on feedback of the zero-tilting moment point (ZMP). Let us look at the working assumptions and components behind it.

If this is your first time reading this page, I warmly advise you watch this excellent documentary from the NHK on ASIMO. It does a good job at explaining some of the key concepts that we are define more formally below.

Linear inverted pendulum model

The common model for (fixed or mobile) robots consists of multiple rigid bodies connected by actuated joints. The general equation of motion for such a system are high-dimensional, but they can be reduced using three working assumptions:

- *Assumption 1:* the robot has enough joint torques to realize its motions.
- *Assumption 2:* there is no angular momentum around the center of mass (CoM).
- *Assumption 3:* the center of mass keeps a constant height.

Assumptions 2 and 3 explain why you see the Honda P2 walk with locked arms and bent knees. Under these three assumptions, the equations of motion of the walking biped are reduced to a linear model, the *linear inverted pendulum*:

$$\ddot{\boldsymbol{p}}_G = \omega^2 (\boldsymbol{p}_G - \boldsymbol{p}_Z)$$

where $\omega^2 = g/h$, g is the gravity constant, h is the CoM height and $\boldsymbol{p}_Z$ is the position of the zero-tilting moment point (ZMP). The constant ω is called the *natural frequency* of the linear inverted pendulum. In this model, the robot can be seen as a point-mass concentrated at G resting on a massless leg in contact with the ground at Z. Intuitively, the ZMP is the point where the robot applies its weight. As a consequence, this point needs to lie inside the contact surface $\mathcal{S}$.

Jurys and committees

- Examiner in admission jury
- Correcteur pour le concours X-ENS INFO B 2023
- Examiner in CSI committees (4×)
- Examiner in PhD juries (2×)

Organization

- Outreach chair in the Organizing Committee for IEEE-RAS Humanoids 2024
- Organizer of the COMANOID locomotion workshop, 2018
- Student helper at the workshop on Robotics and Military Applications: From Current Research and Deployments to Legal and Ethical Questions at ICRA 2014

Teaching

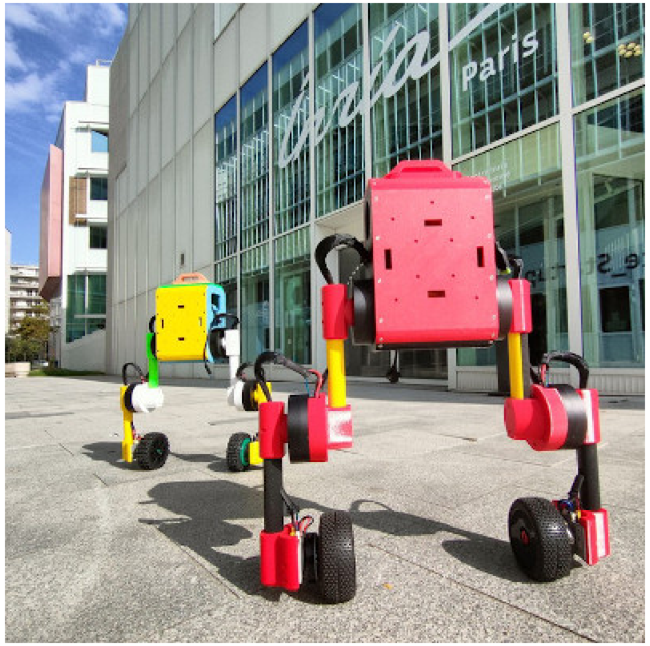
- Robotics class at Master MVA, 18 hours, 2023-present
- Robotics class at École Normale Supérieure, 18 hours, 2023-present

Students

- V. Tordjman--Levavasseur (MSc, 2024)
- U. B. Gökbakan (PhD, 2024), co-supervised with P. Souères
- V. Ledoux (MSc, 2023)
- S. Samadi (PhD year 1, 2019)
- V. Thomasset (PhD years 1-2, 2018-2019)

Open source

- upkie: open-source wheeled-biped robots.
- lipm_walking_controller: humanoid stair climbing.
- pink: QP-based differential IK based on Pinocchio.
- robot_descriptions: collection of open source robot descriptions and software to load them.
- qpsolvers: quadratic programming solvers.
- qpbenchmark: benchmark for QP solvers.



Research projects

I participate or have been involved in the following international research projects:

- 2024-present: PR[AI]RIE-PSAI, Cluster IA Program, France 2030 strategy.
- 2023-present: AGIMUS, next-generation robots for agile production.
- 2016-2019: COMANOID, collaborative humanoids in aircraft manufacturing.
- 2015: DARPA Robotics Challenge, Team Hydra.

Next steps

Vision-guided locomotion policies

How can vision improve locomotion performance?

Intermediate representations:

- Hartley *et al.*, 2018: visuo-inertial legged odometry
- Miki *et al.*, 2020: elevation mapping, height scans
- Loquercio *et al.*, 2022: future terrain heights

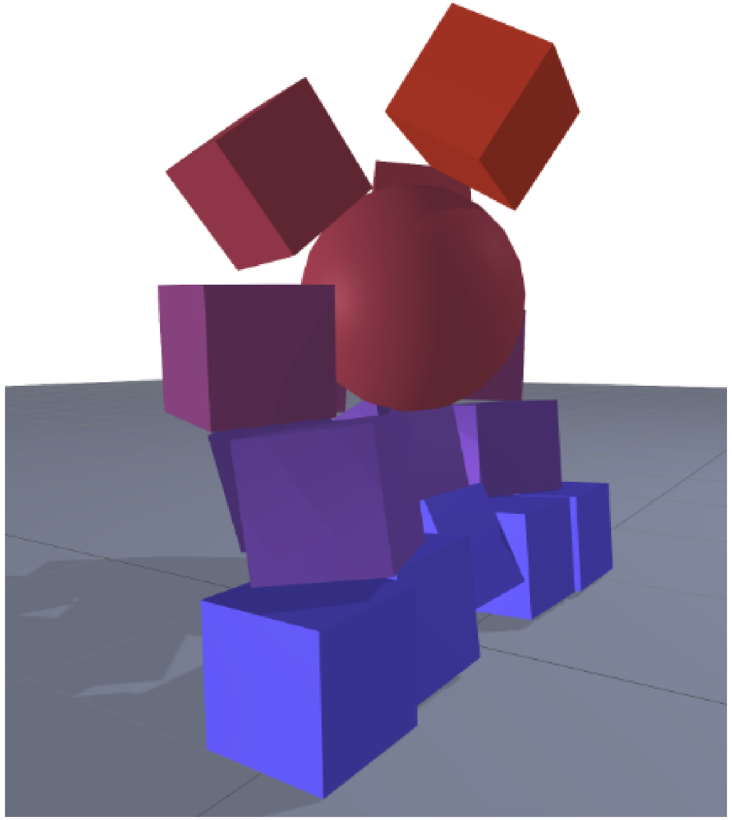
Questions:

- What kind of visual features?
- Effect of head/visual center stabilization?

Differentiable simulation

How can we transfer Real2Sim to a differentiable simulator?

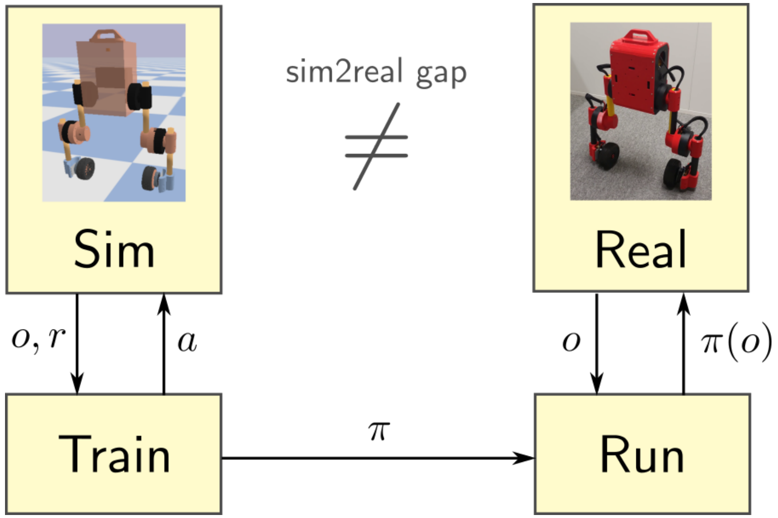
- Le Lidec *et al.*, 2024: diff. simulator developed at Willow
- Differentiable optimal control pipeline: model predictive control using ProxSuite's QPLayer
- What models learned from a given task? How many tasks to learn a given model?



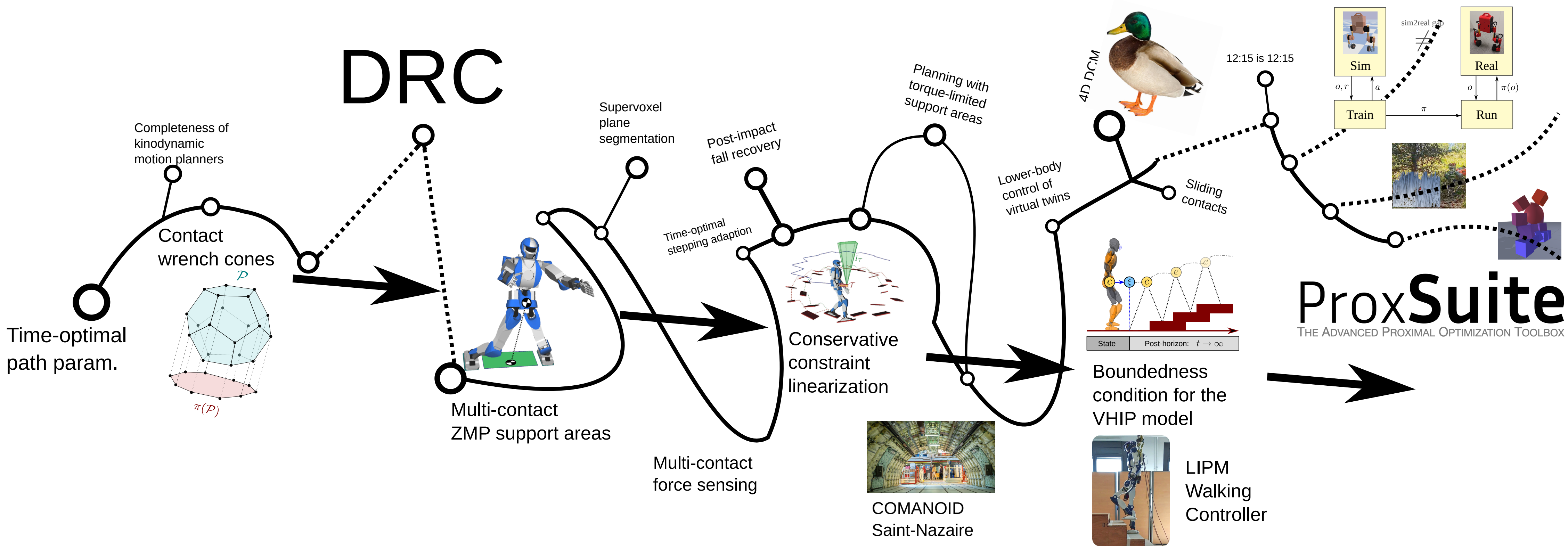
Visual and proprioceptive data

What kind of data can we build on?

- Current practice: on-policy algorithms, reward shaping
- First-order algorithms in a differentiable simulation
- Open X-Embodiment, 2023: aim for large pretrained
- Ghosh *et al.*, 2024: generalist policy, finetuning
- Database of robot experience suited to diff. simulation?



Questions?


DRC
Completeness of kinodynamic motion planners
Contact wrench cones
$\mathcal{P}$
$\pi(\mathcal{P})$
Time-optimal path param.
Supervoxel plane segmentation
Multi-contact ZMP support areas
Post-impact fall recovery
Time-optimal stepping adaption
Multi-contact force sensing
Planning with torque-limited support areas
Conservative constraint linearization
COMANOID Saint-Nazaire
4D DCM
Lower-body control of virtual twins
Sliding contacts
State
Post-horizon: $t \to \infty$
Boundedness condition for the VHIP model
LIPM Walking Controller
12:15 is 12:15
Sim
Real
sim2real gap
o, r
a
o
$\pi(o)$
Train
Run
π
ProxSuite
THE ADVANCED PROXIMAL OPTIMIZATION TOOLBOX