Robotics: Science and Systems
Tuesday, July 14, 2015

Double Description Method

Stability conditions are known locally at contact areas (see Contact Wrench Cone)
Use the Equations of Motion to combine them at the COM:

$$\mathbf{f}_{\mathrm{GI}} = - \sum_{\text{contact } i} \mathbf{R}_i \mathbf{f}_i$$

$$\boldsymbol{\tau}_{\mathrm{GI}} = - \sum_{\text{contact } i} (\mathbf{p}_i - \mathbf{p}_{\mathrm{CoM}}) \times \mathbf{R}_i \mathbf{f}_i + \mathbf{R}_i \boldsymbol{\tau}_i$$

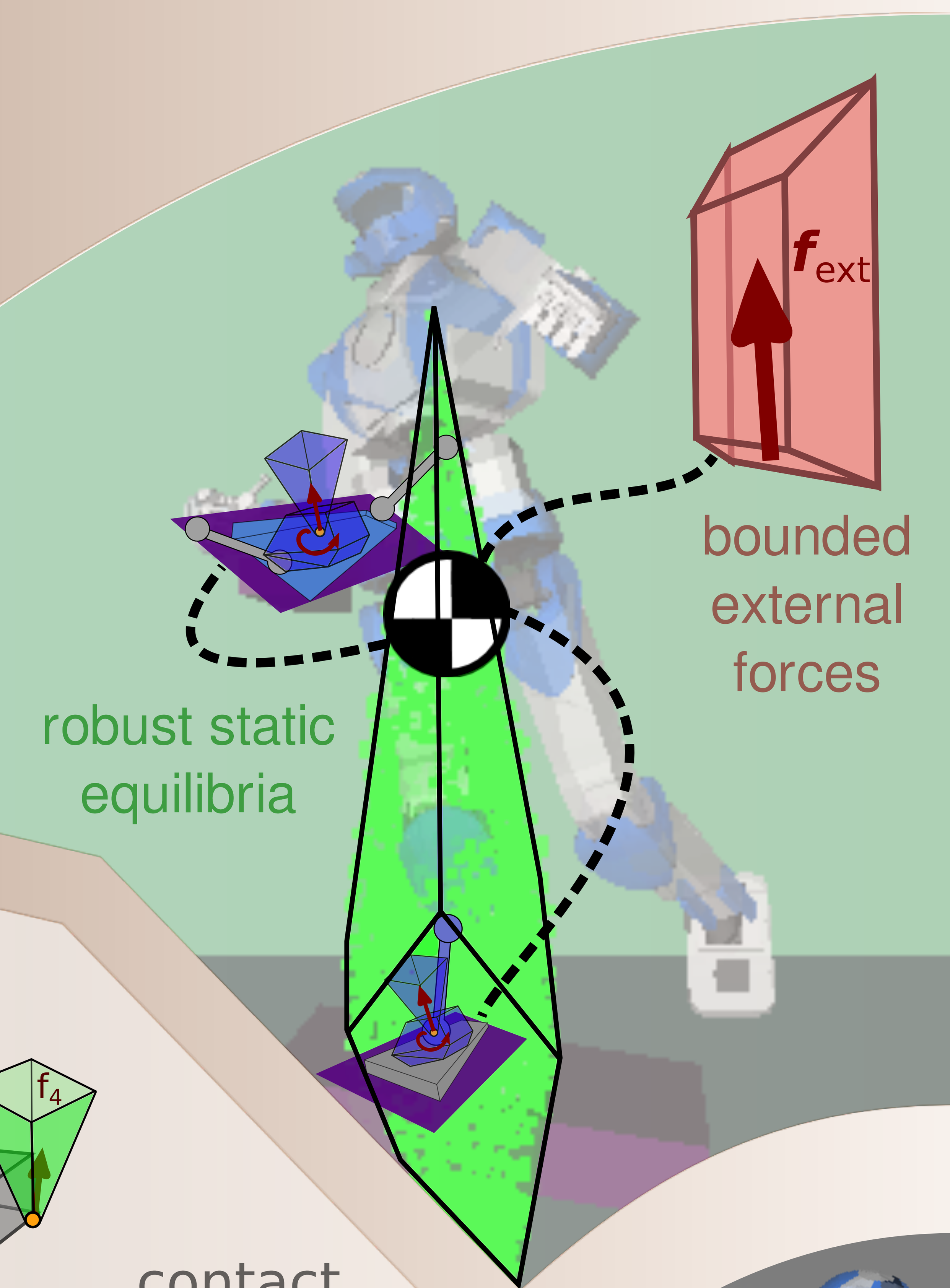
Gravito-Inertial Wrench

$$\mathbf{f}_{\mathrm{GI}} := m(\mathbf{g} - \ddot{\mathbf{p}}_{\mathrm{CoM}})$$

$$\boldsymbol{\tau}_{\mathrm{GI}} := \mathbf{p}_{\mathrm{CoM}} \times m(\mathbf{g} - \ddot{\mathbf{p}}_{\mathrm{CoM}}) - \dot{\mathcal{L}}$$

Apply a Cone Duality algorithm to convert local polytopes $(\mathbf{f}_i, \boldsymbol{\tau}_i) \in \mathcal{CWC}_i$
to an equivalent 6D polytope $(\mathbf{f}_{\mathrm{GI}}, \boldsymbol{\tau}_{\mathrm{GI}}) \in \mathcal{GIWC}$,
the Gravito-Inertial Wrench Cone.

Fastest known: *Double-Description Method* (Motzkin et al.)
Code: cddlib (C), pycddlib (Python), ...



Contact Wrench Cone

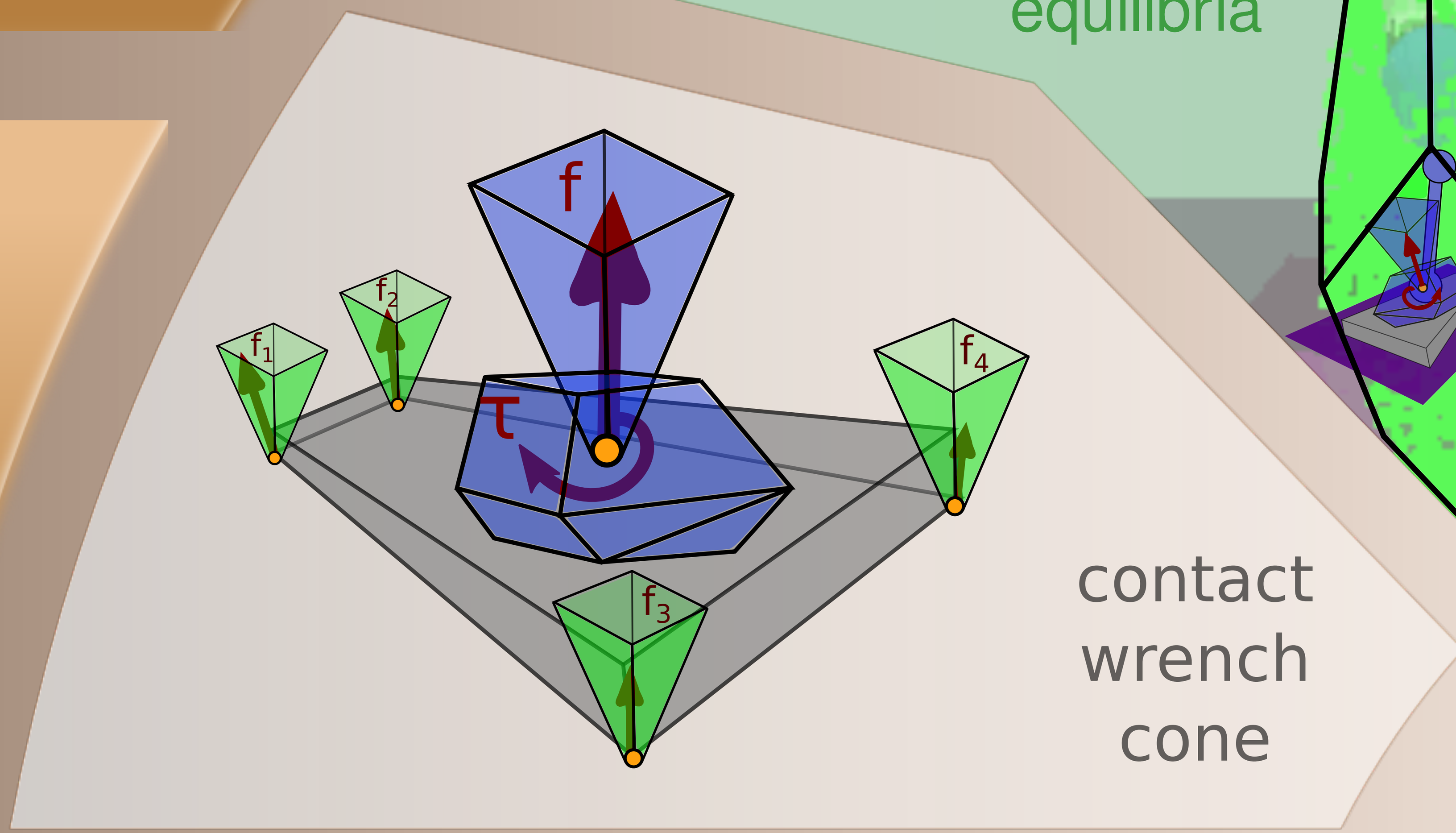
Same physical model as contact force distribution
Structure : Friction cone + COP area + Yaw bounds
Symbolic calculation (Fourier-Motzkin algorithm)

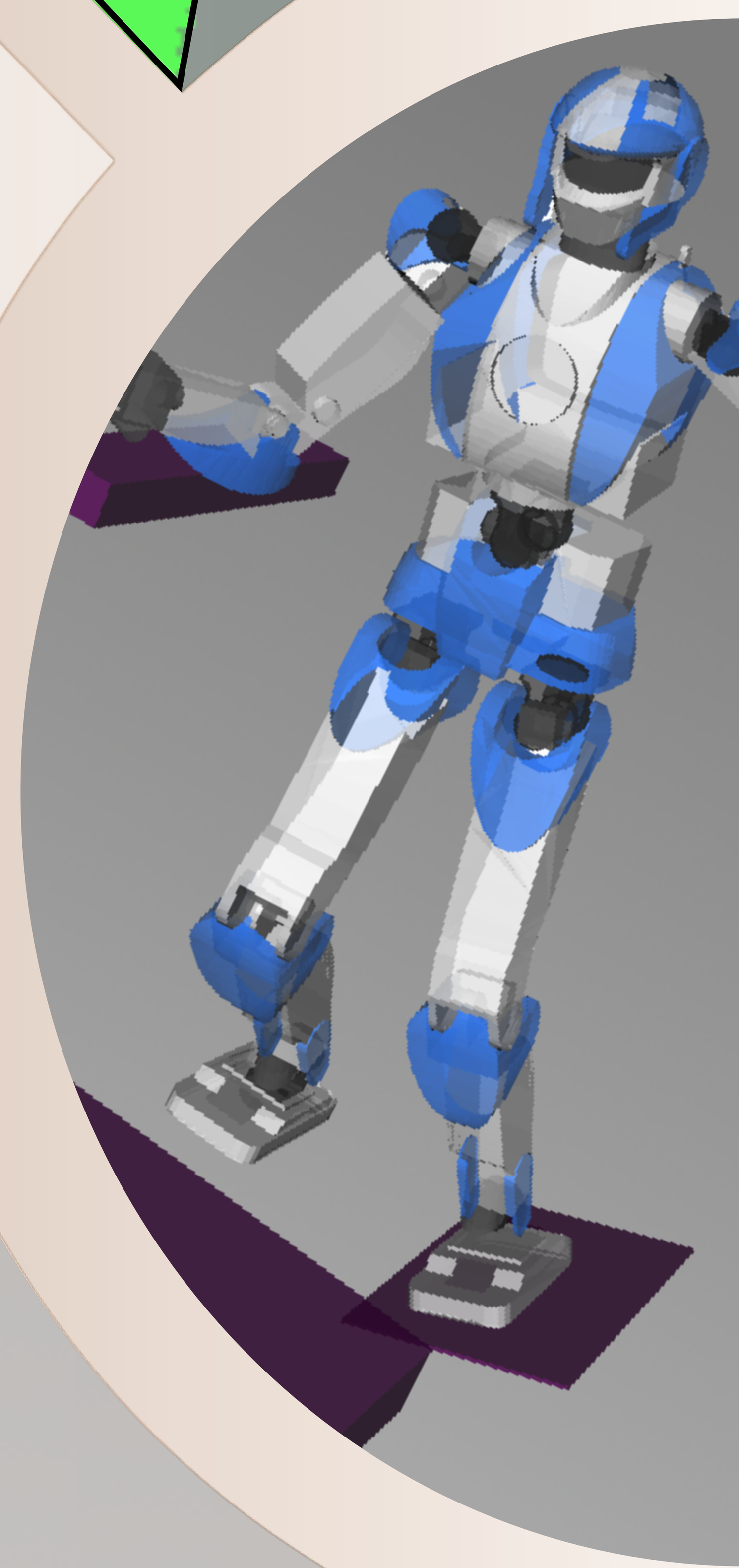
$$f^z > 0 \qquad \tau_{\min}(\boldsymbol{f}, \tau^x, \tau^y) \leq \tau^z \leq \tau_{\max}(\boldsymbol{f}, \tau^x, \tau^y)$$

$$\|\boldsymbol{f}^t\| \leq \mu f^z$$

$$|\tau^y| \leq X f^z \qquad \tau_{\min}: \quad \mu(X+Y)f^z + Yf^x \quad \mu\tau^x + Xf^y \quad \mu\tau^y$$

$$|\tau^x| \leq Y f^z \qquad \tau_{\max}: \quad +\mu(X+Y)f^z \quad Yf^x + \mu\tau^x \quad Xf^y + \mu\tau^y$$





Leveraging Cone Double Description for Multi-Contact Stability of Humanoids with Applications to Statics and Dynamics

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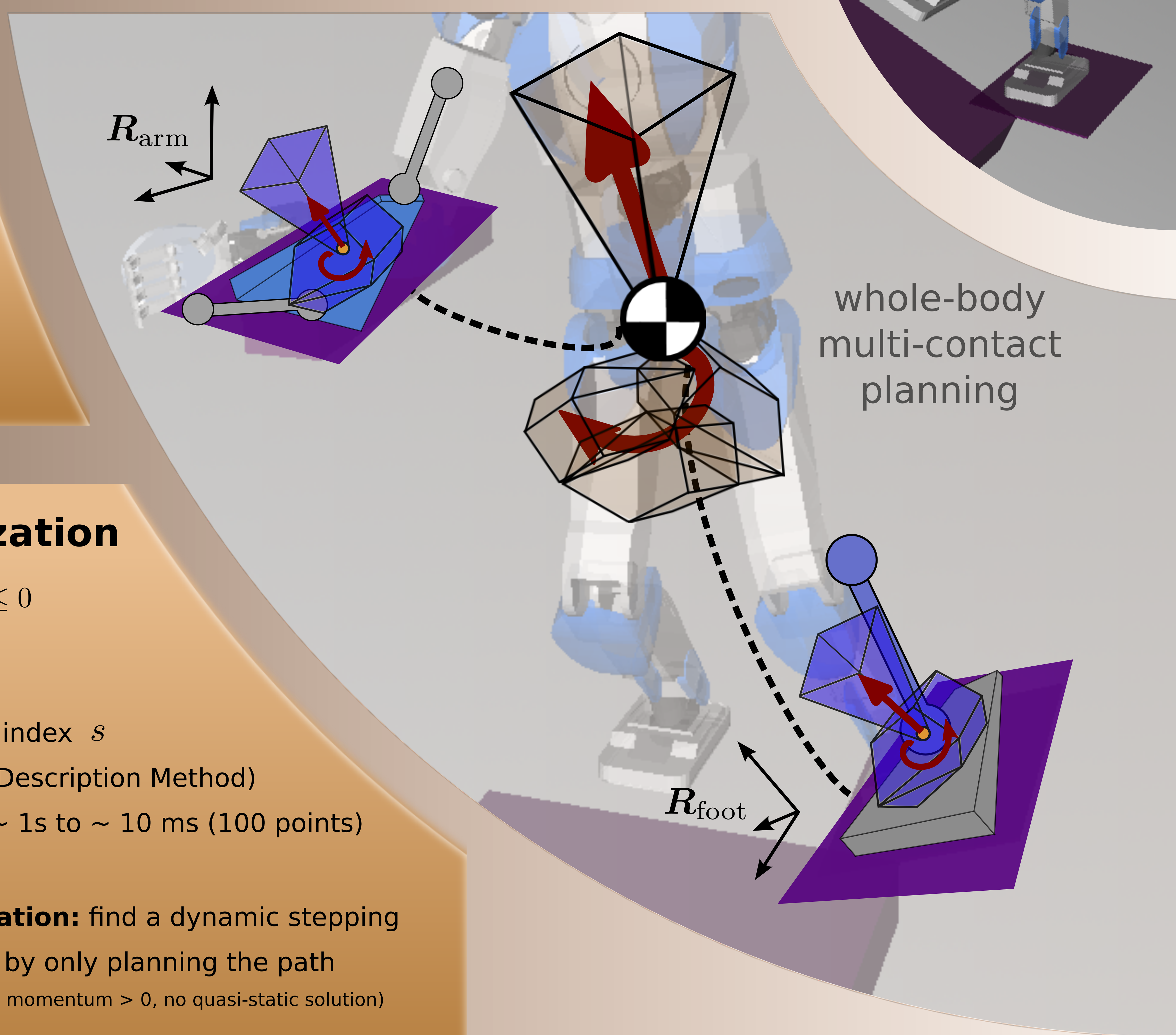
Robust Static Equilibrium

Goal: resist *any* bounded external force
Model: $\boldsymbol{f}_{\text{ext}}$ bounded and applied at COM
Solution: polytope $\mathcal{P}$ of COM positions
Prop: can resist any $\boldsymbol{f}_{\text{ext}}$ iff COM inside $\mathcal{P}$
Computing $\mathcal{P}$: ~1 ms by Double Description Method

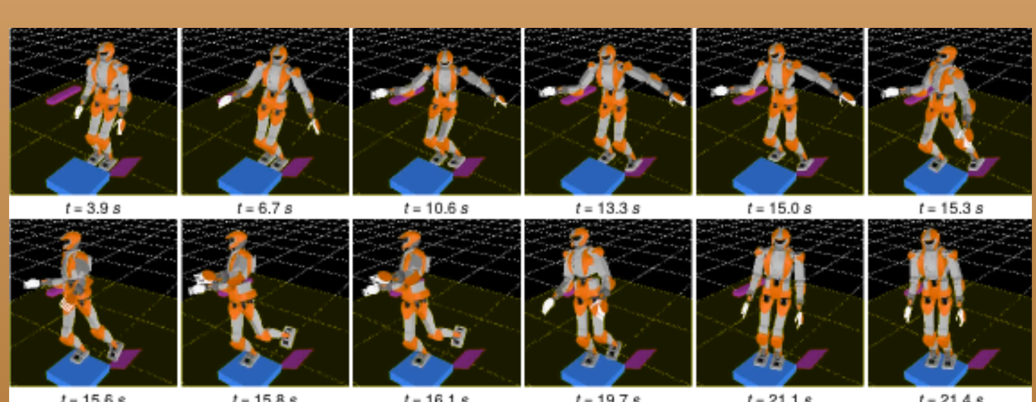


Time-Optimal Path Parameterization

Gravito-Inertial Wrench Cone: $\mathbf{A}(\mathbf{p}_{\mathrm{CoM}})[\mathbf{f}_{\mathrm{GI}}\ \boldsymbol{\tau}_{\mathrm{GI}}]^{\mathrm{T}} \leq 0$
Retiming a path $\mathbf{p}_{\mathrm{CoM}}(s)$ is possible for constraints

$$\ddot{s}\mathbf{a}(s) + \dot{s}^2\mathbf{b}(s) + \mathbf{c}(s) \leq 0$$

Previous approach: polytope projection at each path index s
Contribution: compute the projector matrix (Double-Description Method)
Time to compute all $(\mathbf{a}(s), \mathbf{b}(s), \mathbf{c}(s))$: reduction from ~ 1s to ~ 10 ms (100 points)



Application: find a dynamic stepping motion by only planning the path
(minimum momentum > 0, no quasi-static solution)