

# Kinodynamic Motion Planners based on Velocity Interval Propagation

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RSJ 2013 – IS5 on Humanoid Robots

# Outline

- ▶ Reminder on Randomized Planning
- ▶ Admissible Velocity Propagation algorithm
- ▶ Preliminary experiments
- ▶ Towards humanoid robots...

# Kinodynamic planning

- ▶ **Non-holonomic constraint:**

$$\ddot{q} = f(q, \dot{q}, \tau)$$

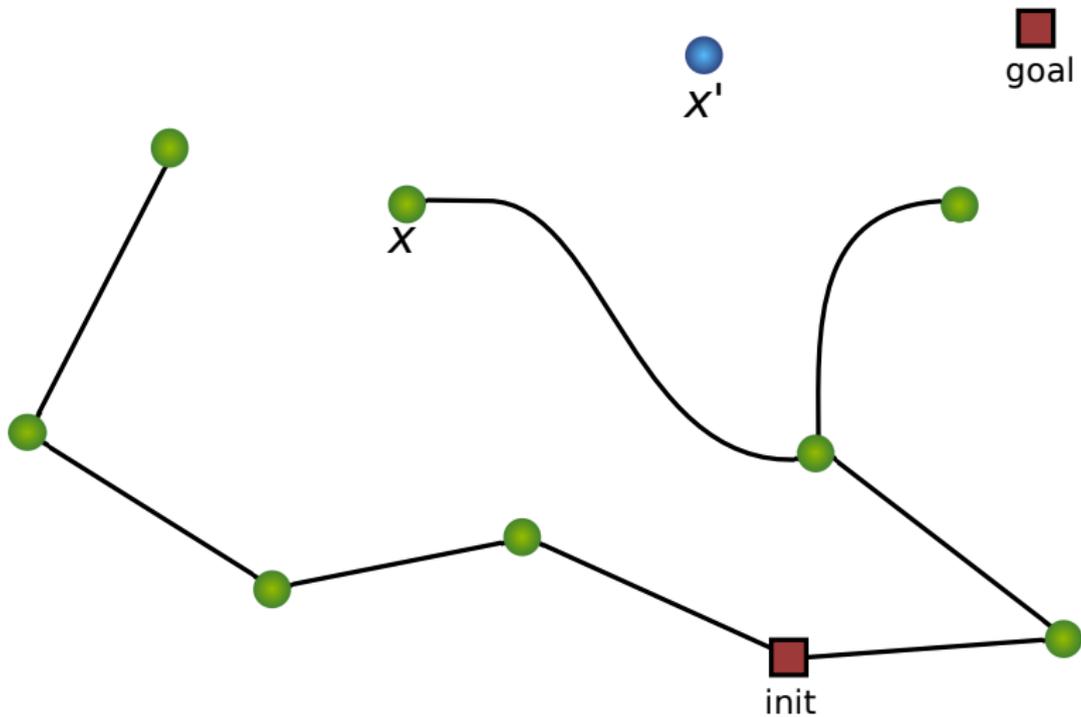
- ▶ **Torque constraints:** for every joint  $i$ ,

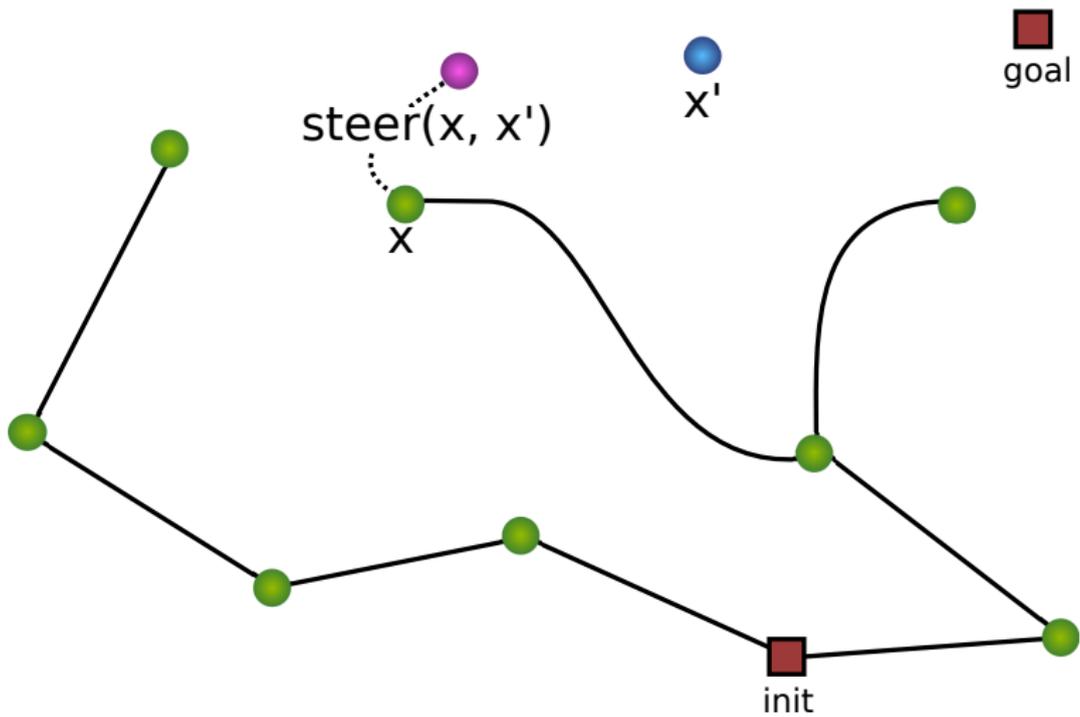
$$|\tau_i| \leq \tau_i^{\max}$$

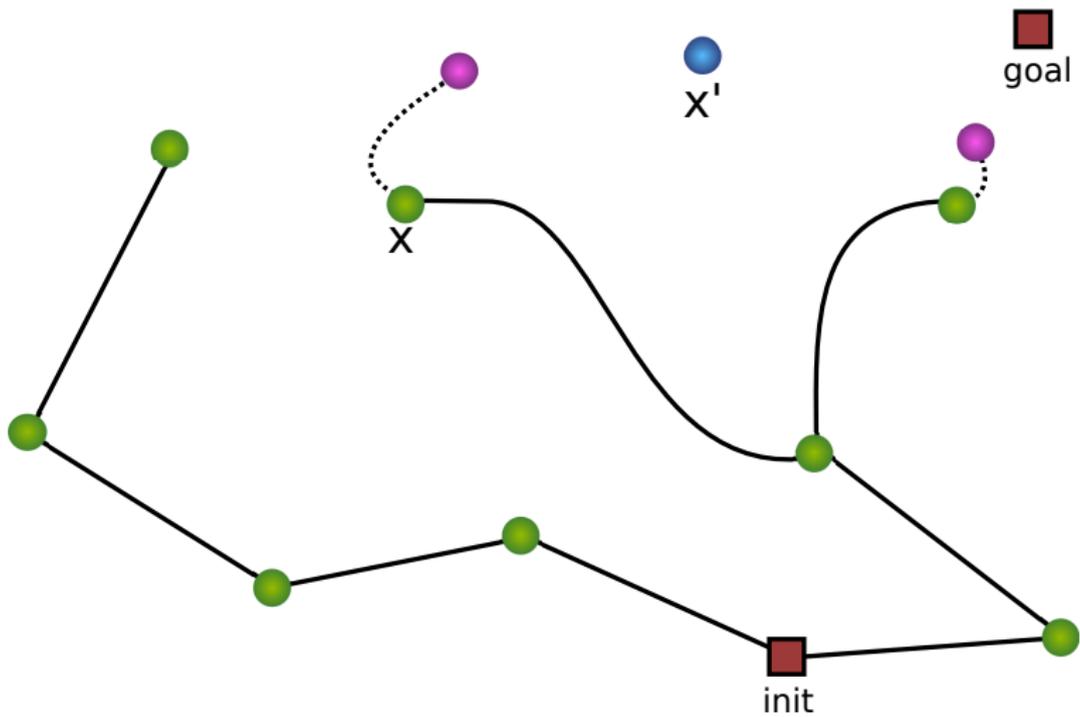
# Randomized motion planning

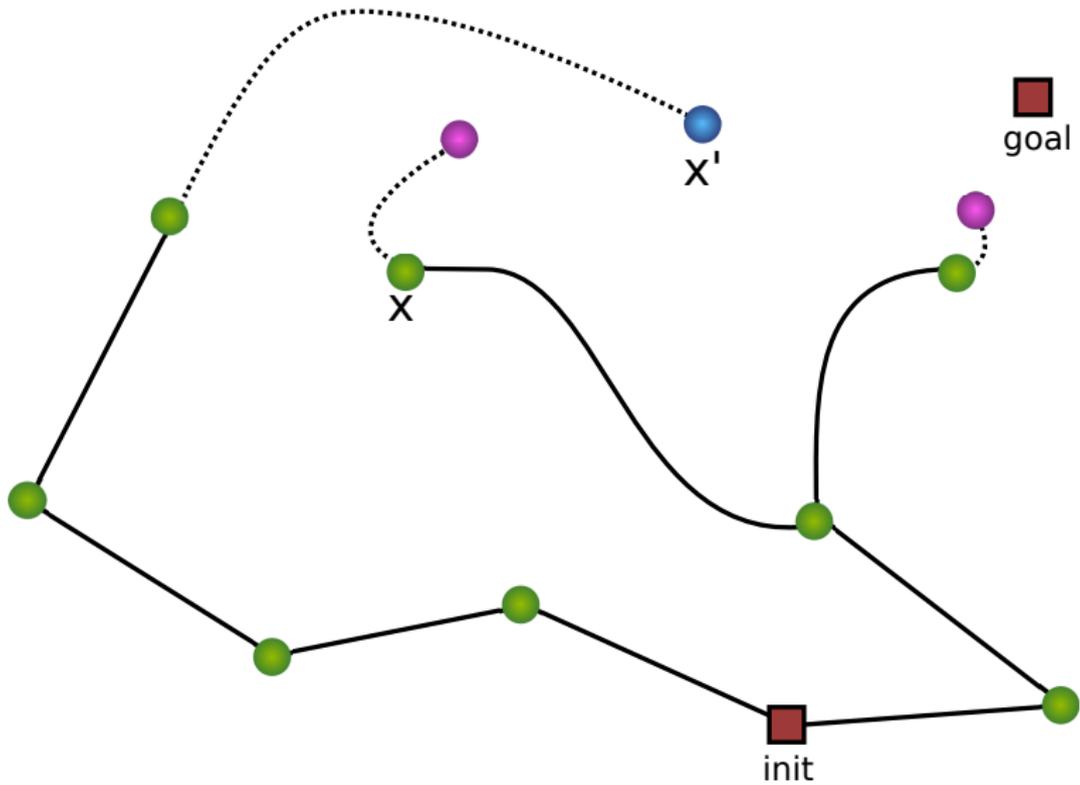
- ▶ Major algorithms:
  - Probabilistic Roadmap (PRM)
  - Rapidly-expanding Random Tree (RRT)
- ▶ **Pro:** probabilistic completeness guarantee (established for kinematic planning)
- ▶ **Con:** curse of dimensionality











# Requirements

- ▶ **Steering function**

steer( $x, x'$ ): reachable state closer to  $x'$

- ▶ **Antecedent search:**

finding nodes to steer *from*

In kinematic planning:

- ▶ steering: geometric interpolation
- ▶ antecedent: neighborhoods for a metric  $\sigma(x, x')$

What about kinodynamic planning?

# Steering

- ▶ Forward dynamics based (non-humanoid)  
[LaValle, 1998, Hsu et al., 2002]
- ▶ Optimal steering (non-humanoid)  
[Karaman and Frazzoli, 2011]
- ▶ **Inverse dynamics based** [Kuffner et al., 2002]

# Steering with inverse dynamics?

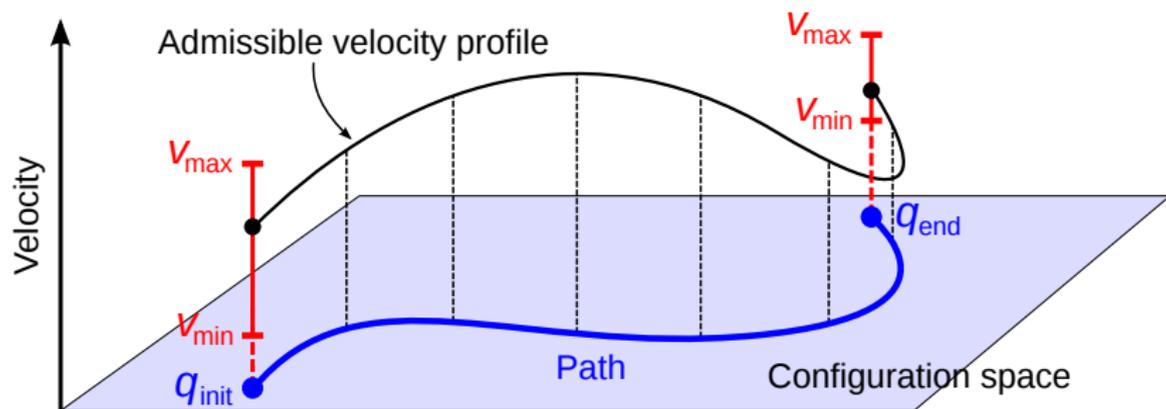
- ▶ **Previous approach:**
  - interpolate a *trajectory*
  - apply some dynamics filter [Kuffner et al., 2002]
- ▶ **Our approach:**
  - interpolate a *path*
  - propagate reachable-velocity intervals [Pham et al., 2013]

# Admissible Velocity Propagation

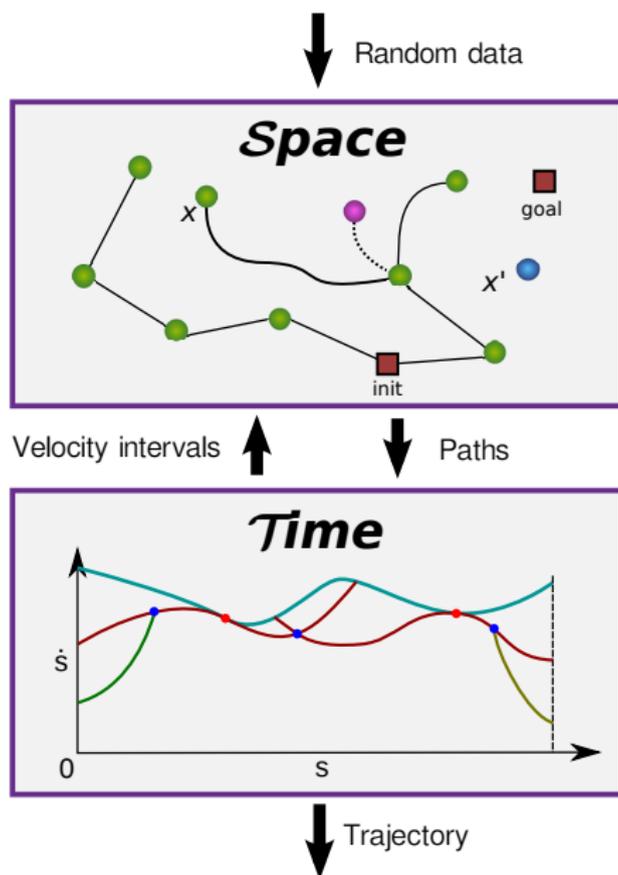
- ▶ **AVP algorithm:** extension of the Time-Optimal Path Tracking algorithm [Bobrow et al., 1985]
- ▶ **Input:**
  - path  $P \subset \mathcal{C}_{\text{free}}$
  - interval of admissible velocities  $[v_{\min}^{\text{init}}, v_{\max}^{\text{init}}]$
- ▶ **Output:**
  - is the path traversable?
  - interval of reachable velocities  $[v_{\min}^{\text{end}}, v_{\max}^{\text{end}}]$

# Planner integration

- ▶ Each node stores a state  $x$  and a velocity interval  $[v_{\min}, v_{\max}]$
- ▶ Extension: interpolate a path, propagate admissible velocities



# Space $\times$ time decoupling



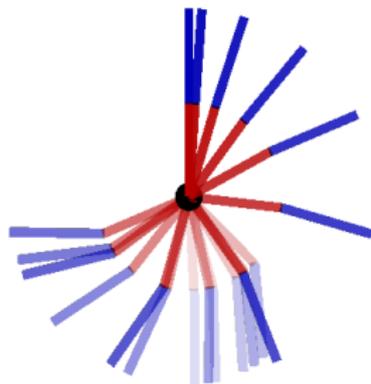
# Properties

- ▶ Initial path unchanged  $\rightarrow$  collision checking
- ▶ Applies to second-order non-holonomic constraints:  
ZMP balance, torque limits, ...



Figure: Screenshot from [Pham and Nakamura, 2012]

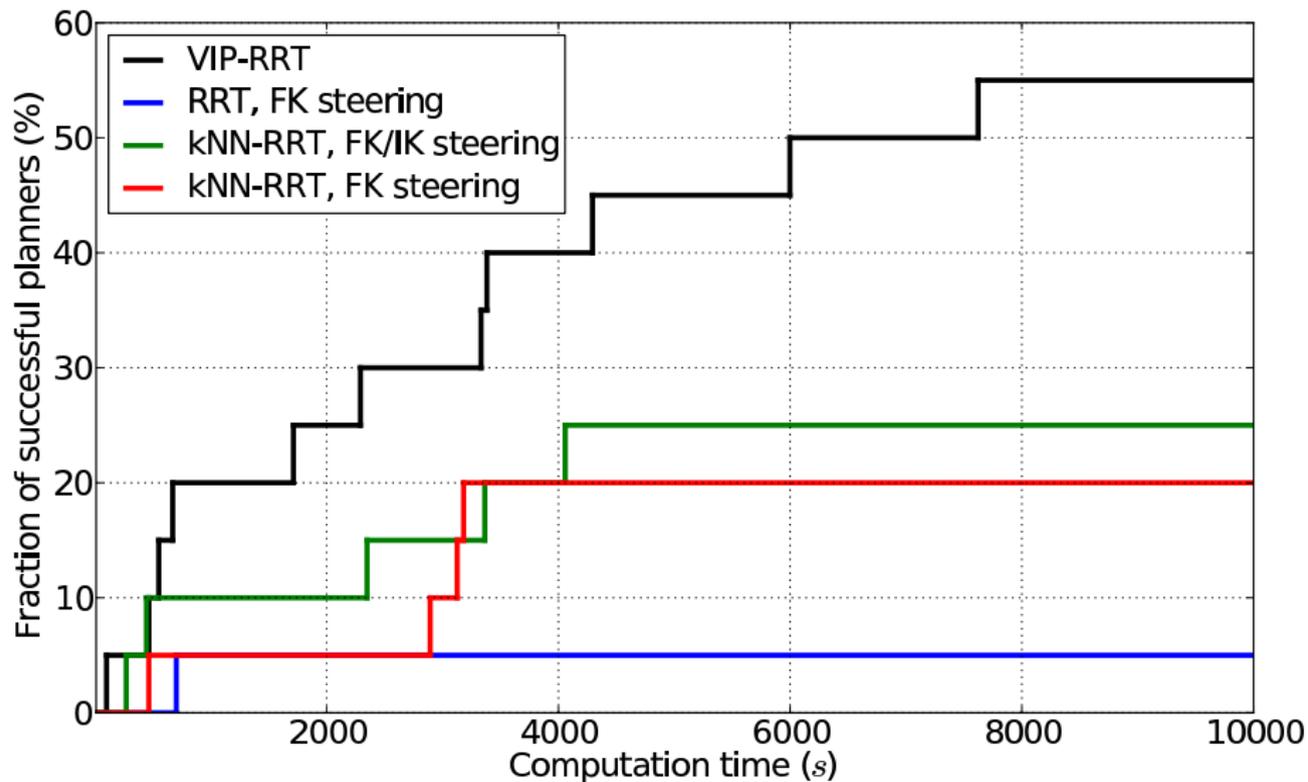
# Preliminary experiments



## Double-inverted pendulum:

- ▶ Link: length  $l = 0.2$  m
- ▶ Link mass  $m = 1$  kg
- ▶ Statically-stable planning:  $|\tau_1| > 15.6$  N.m
- ▶ Torque limits:  $|\tau_1| \leq 8$  N.m  $\wedge$   $|\tau_2| \leq 4$  N.m

# Simulation results



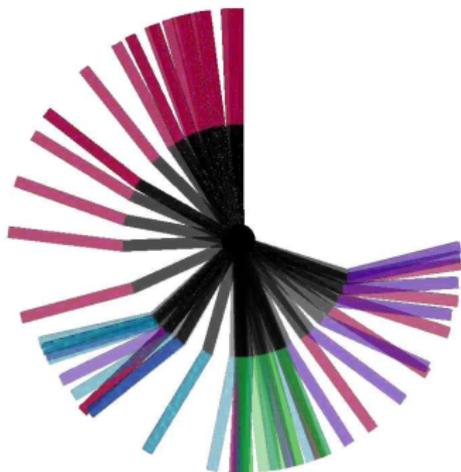
# Towards Humanoids

- ▶ Extension to under-actuated systems: decoupling vector fields [Bullo and Lynch, 2001]
- ▶ Identifying actuator limits
- ▶ ...

# To be continued...

- ▶ Randomized **kinodynamic** planning for humanoids?
- ▶ Importance of **steering** and **antecedent selection**
- ▶ Our approach steering: path tracking with **velocity interval propagation**

Thanks for your attention!



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