Pendular models for walking over rough terrains

Stéphane Caron

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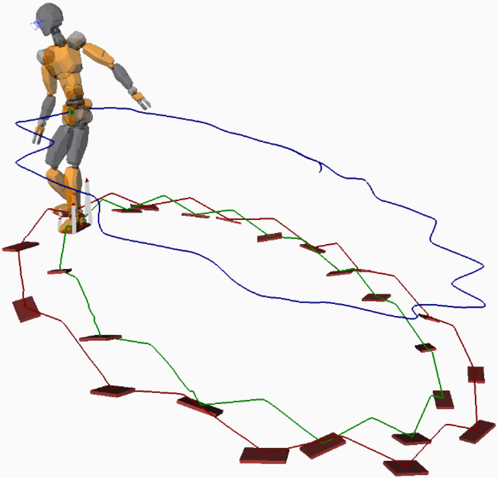
Journées Nationales de la Robotique Humanoïde, Montpellier, France

GOAL



Standard model reduction

Equation of motion

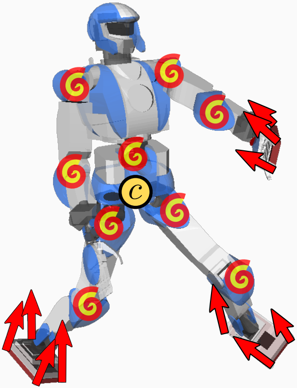
$$M\ddot{q} + h(q, \dot{q}) \; = \; S^T\tau + J_c^T F$$

Constraints

- $\tau \in$ {feasible torques}
- $F \in$ {feasible contact forces}

Assumption

- (Rigid bodies)



Equations of motion

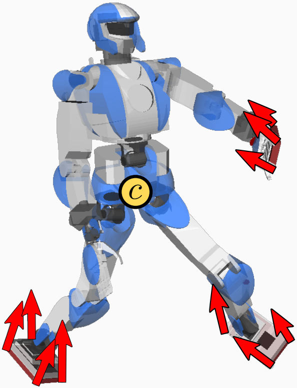
$$\ddot{c} = \frac{1}{m}\sum_i f_i + \vec{g}$$
$$\dot{L}_c = \sum_i (p_i - c) \times f_i$$

Constraints

- Friction cones: $\forall i, f_i \in \mathcal{C}_i$

Assumption

- Infinite torques

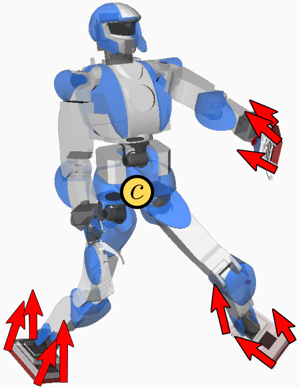


Equations of motion

$$\ddot{c} = \frac{1}{m}\sum_i f_i + \vec{g}$$

$$\dot{L}_c = \sum_i (p_i - c) \times f_i$$

Forward integration **approximated** by iterative methods (e.g. RK4)

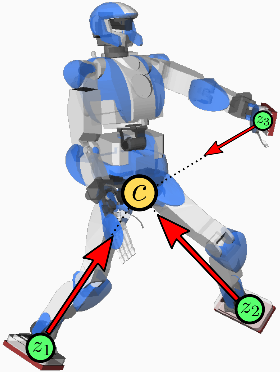


Pendular mode

$$\dot{L}_c = 0$$

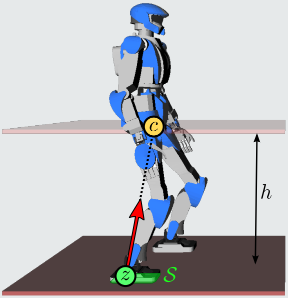
Conserve the angular momentum at the center-of-mass

- **Pro:** enables exact forward integration
- **Con:** assumes $\dot{L}_c = 0$ feasible regardless of joint state



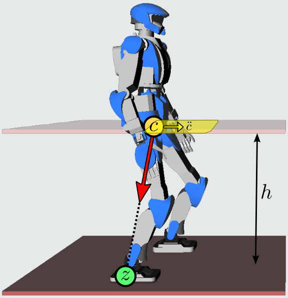
From 2D to 3D locomotion

LIPM [Kaj+01]



- Control: $z \in \mathcal{S}$
- Output: $\ddot{c}$

CART-table [Kaj+03]



- Control: $\ddot{c} \in \omega^2(c - \mathcal{S}) + \vec{g}$
- Output: z

Equation of motion

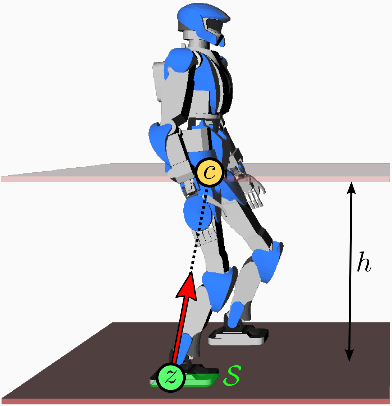
$$\ddot{c} \;=\; \omega^2(c - z) + \vec{g}$$

Constraints

- ZMP support area: $z \in \mathcal{S}$

Assumptions

- Infinite torques
- Pendular mode
- COM lies in a plane: $c_z = h$
- *Infinite friction*
- *Contacts are coplanar*



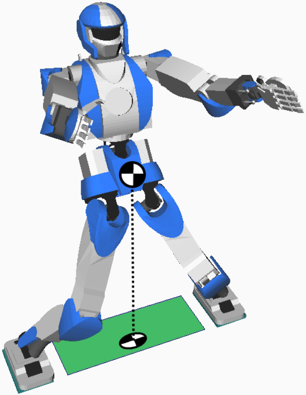


Figure 1: ZMP support area with friction [CPN17]

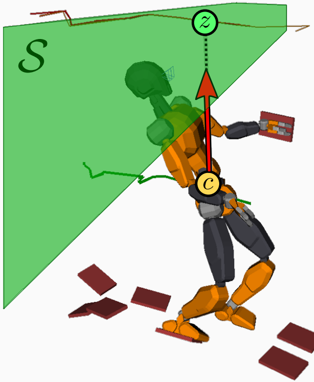


Figure 2: ZMP support area with non-coplanar contacts [CPN17]

Equation of motion

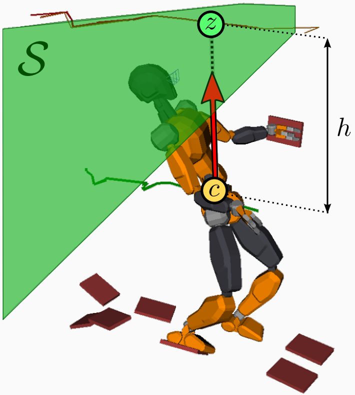
$$\ddot{c} \; = \; \pm\omega^2(c - z) + \vec{g}$$

Constraints

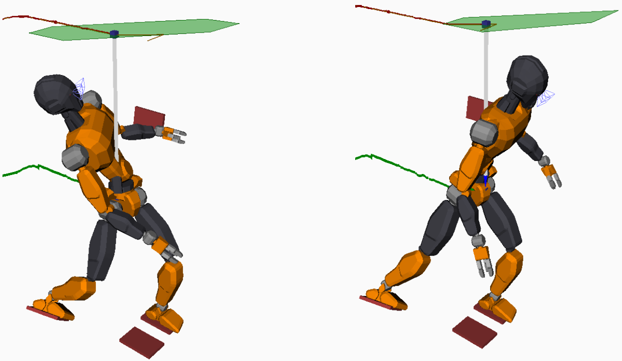
- ZMP support area: $z \in \mathcal{S}$

Assumptions

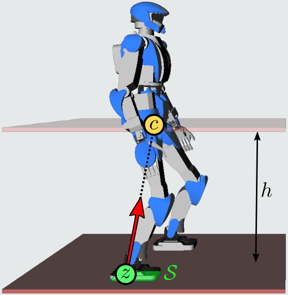
- Infinite torques
- Pendular mode
- *COM lies in a virtual plane* chosen via $\pm\omega^2 = g/h$



ZMP support area $\mathcal{S}$ changes with COM position:

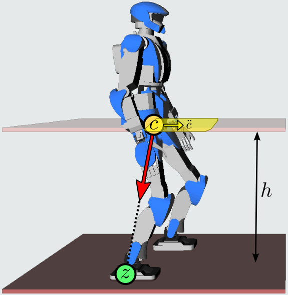


2D LIPM



- Control: $z \in \mathcal{S}$
- Output: $\ddot{c}$

2D CART-table



- Control: $\ddot{c} \in \omega^2(c - \mathcal{S}) + \vec{g}$
- Output: z

3D CART-table

Algorithm [CK16]

Compute the 3D cone $\mathcal{C}$ of COM accelerations

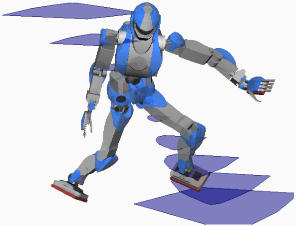


Figure 3: ZMP support areas for different values of $\pm\omega^2$

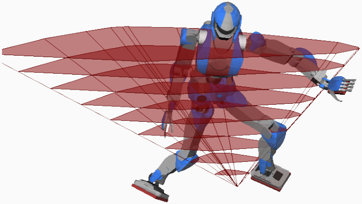
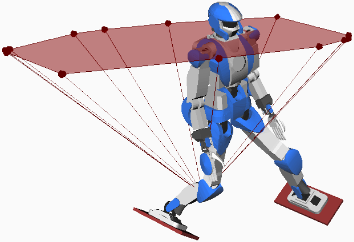
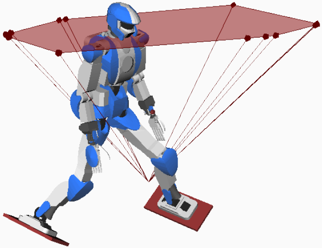


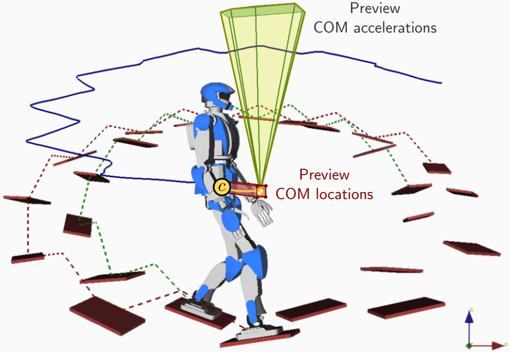
Figure 4: COM acceleration cone for the same stance

The cone $\mathcal{C}$ still depends on the COM position c:

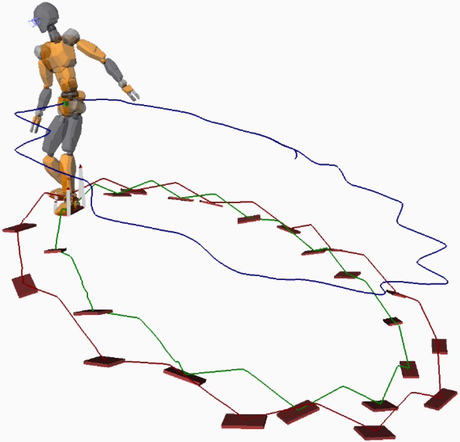




For predictive control, intersect cones $\mathcal{C}$ over all c $\in$ preview:



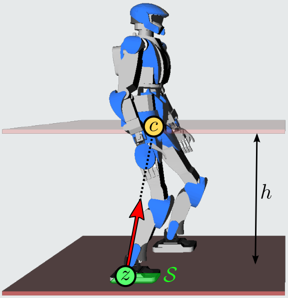
Walking patterns *not very dynamic*, but works surprisingly well!



https://github.com/stephane-caron/3d-com-mpc

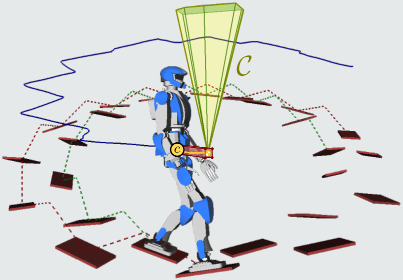
3D Pendulum Mode

2D LIPM



- Control: $z \in \mathcal{S}$
- Output: $\ddot{c}$

3D COM-accel [CK16]



- Control: $\ddot{c} \in \mathcal{C}(c)$
- Output: z

Linear Inverted Pendulum

$$\ddot{c} \;=\; \omega^2(c - z) + \vec{g}$$

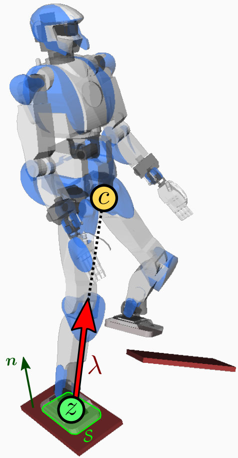
Plane assumption: $\omega = \sqrt{\frac{g}{h}}$

$$\downarrow$$

Remove this assumption:

Inverted Pendulum

$$\ddot{c} \;=\; \lambda(c - z) + \vec{g}$$



Equation of motion

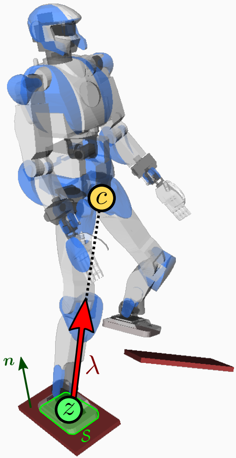
$$\ddot{c} = \lambda(c - z) + \vec{g}$$

Constraints

- Unilaterality $\lambda \geq 0$
- ZMP support area: $z \in \mathcal{S}$

Assumptions

- Infinite torques
- *Infinite friction*
- Pendular mode



Equation of motion

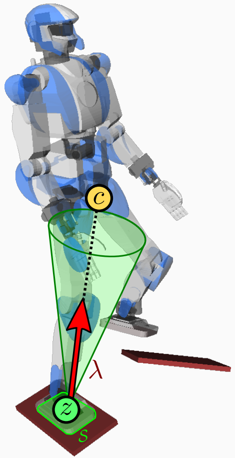
$$\ddot{c} = \lambda(c - z) + \vec{g}$$

Constraints

- Unilaterality $\lambda \geq 0$
- ZMP support area: $z \in \mathcal{S}$
- Friction: $c - z \in \mathcal{C}$

Assumptions

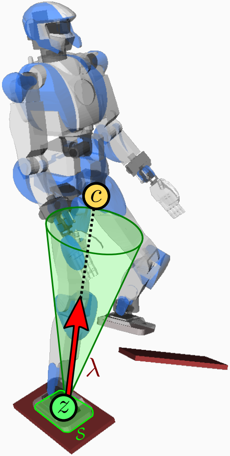
- Infinite torques
- Pendular mode



Equation of motion

$$\ddot{c} \;=\; \lambda(c - z) + \vec{g}$$

- Product bwn control and state
- Forward integration: how to make it **exact**?



Floating-base inverted pendulum (FIP)

Allow the ZMP to leave the contact area.[1]

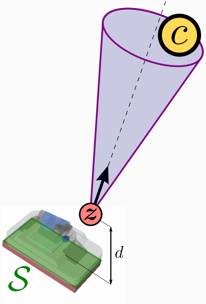


Figure 5: Friction constraint

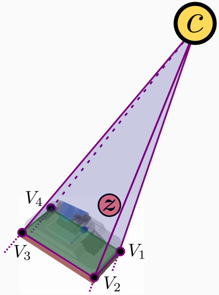


Figure 6: ZMP constraint

[1] At heart, it is used to locate the central axis of the contact wrench [SB04]

Equation of motion

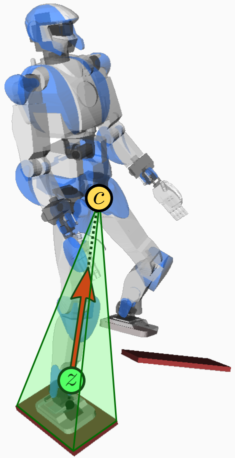
$$\ddot{c} = \omega^2(c - z) + \vec{g}$$

Constraints [CK17]

- Friction: $c - z \in \mathcal{C}$
- ZMP support cone:
  $\forall i, e_i \cdot (v_i - c) \times (z - v_i) \leq 0$

Assumptions

- Infinite torques
- Pendular mode



Equation of motion

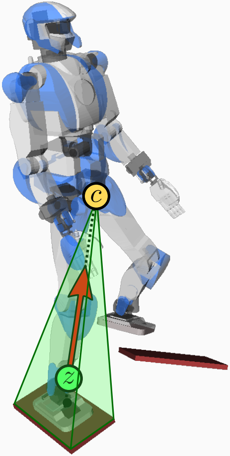
$$\ddot{c} \; = \; \omega^2(c - z) + \vec{g}$$

- Forward integration is **exact**:

$$c(t) = \alpha_0 e^{\omega t} + \beta_0 e^{-\omega t} + \gamma_0$$

- Capture Point is defined:

$$\xi = c + \frac{\dot{c}}{\omega} + \frac{\vec{g}}{\omega^2}$$

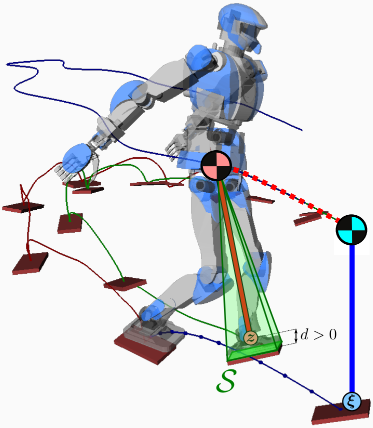


NMPC Optimization

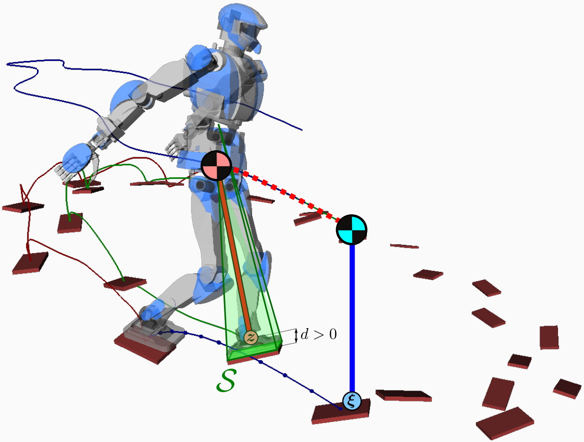
- Runs at 30 Hz
- Adapts step timings
- FIP for forward integration
- Sometimes fails...

Linear-Quadratic Regulator

- Runs at 300 Hz
- Takes over when NMPC fails



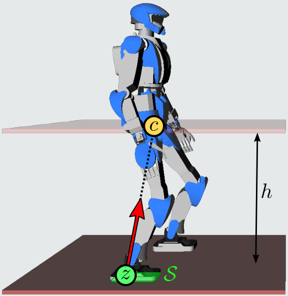
CHECK IT OUT!



https://github.com/stephane-caron/dynamic-walking

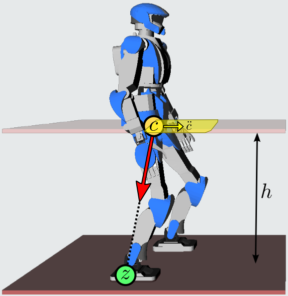
CONCLUSION

2D LIPM



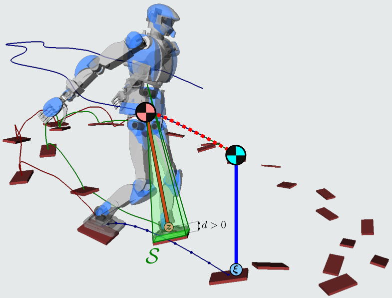
- Control: $z \in \mathcal{S}$
- Output: $\ddot{c}$

2D CART-table



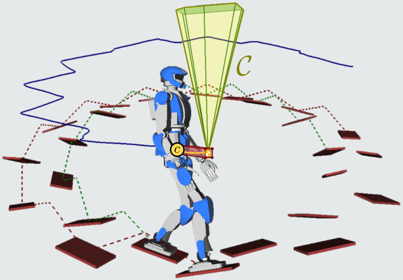
- Control: $\ddot{c} \in \omega^2(c - \mathcal{S}) + \vec{g}$
- Output: z

3D FIP [CK17]



- Control: $z \in \mathcal{S}(c)$
- Output: $\ddot{c}$

3D COM-accel [CK16]



- Control: $\ddot{c} \in \mathcal{C}(c)$
- Output: z

THANKS FOR LISTENING!

[CK16] Stéphane Caron and Abderrahmane Kheddar. “Multi-contact Walking Pattern Generation based on Model Preview Control of 3D COM Accelerations”. In: *Humanoid Robots, 2016 IEEE-RAS International Conference on*. Nov. 2016.

[CK17] Stéphane Caron and Abderrahmane Kheddar. “Dynamic Walking over Rough Terrains by Nonlinear Predictive Control of the Floating-base Inverted Pendulum”. In: *Intelligent Robots and Systems (IROS), 2017 IEEE/RSJ International Conference on*. to be presented. Sept. 2017.

[CPN17] Stéphane Caron, Quang-Cuong Pham, and Yoshihiko Nakamura. “ZMP Support Areas for Multi-contact Mobility Under Frictional Constraints”. In: *IEEE Transactions on Robotics* 33.1 (Feb. 2017), pp. 67–80.

[Kaj+01] Shuuji Kajita, Fumio Kanehiro, Kenji Kaneko, Kazuhito Yokoi, and Hirohisa Hirukawa. “The 3D Linear Inverted Pendulum Mode: A simple modeling for a biped walking pattern generation”. In: *Intelligent Robots and Systems, 2001.* Vol. 1. IEEE. 2001, pp. 239–246.

[Kaj+03] Shuuji Kajita, Fumio Kanehiro, Kenji Kaneko, Kiyoshi Fujiwara, Kensuke Harada, Kazuhito Yokoi, and Hirohisa Hirukawa. “Biped walking pattern generation by using preview control of zero-moment point”. In: *IEEE International Conference on Robotics and Automation*. Vol. 2. IEEE. 2003, pp. 1620–1626.

[SB04] P. Sardain and G. Bessonnet. “Forces acting on a biped robot. center of pressure-zero moment point”. In: *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans* 34.5 (2004), pp. 630–637.