Dynamic Walking Over Rough Terrains by Nonlinear Predictive Control of the Floating-Base Inverted Pendulum

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September 27, 2017

IROS 2017, Vancouver, Canada
Equation of motion

\[ \ddot{c} = \omega^2 (c - z) + \ddot{g} \]

Feasibility conditions

- Constant: \( \omega = \sqrt{g/h} \)
- ZMP support area: \( z \in S \)
- Friction?
### Linear Inverted Pendulum

\[
\ddot{c} = \omega^2 (c - z) + g
\]

### Feasibility conditions

- Constant: \( \omega = \sqrt{g/h} \)
- ZMP support area: \( z \in S \)
- Friction?

### Inverted Pendulum

\[
\ddot{c} = \lambda (c - z) + g
\]

### Feasibility conditions

- Unilaterality: \( \lambda \geq 0 \)
- ZMP support area: \( z \in S \)
- Friction?
Equation of motion

\[ \ddot{c} = \lambda(c - z) + \ddot{g} \]

Feasibility conditions

- **Unilaterality:** \( \lambda \geq 0 \)
- **ZMP support area:** \( z \in S \)
- **Friction?**
Equation of motion

\[ \ddot{c} = \lambda (c - z) + \ddot{g} \]

Feasibility conditions

- Unilaterality: \( \lambda \geq 0 \)
- ZMP support area: \( z \in S \)
- Friction?
### Equation of motion

\[ \ddot{c} = \lambda(c - z) + \ddot{g} \]

### Feasibility conditions

- **Unilaterality:** \( \lambda \geq 0 \)
- **ZMP support area:** \( z \in S \)
- **Friction:** \( c - z \in \mathcal{C} \)
**Equation of motion**

\[ \ddot{c} = \lambda(c - z) + \ddot{g} \]

**Forward integration**

- Direct multiple shooting
- Discretization: # of sample points, integration step
- Resolution of integrator?

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1 See also Takasugi et al. (this session): "3D Walking and Skating..."

2 Carpentier, Tonneau, Naveau, Stasse, and Mansard 2016.
Equation of motion

$$\ddot{c} = \omega^2 (c - z) + \ddot{g}$$

Virtual Repellent Points

- The ZMP/eCMP/VRP can leave the contact area
- Fwd integration is exact:
  $$c(t) = a e^{\omega t} + b e^{-\omega t} + \gamma$$
- Feasibility conditions?

\(^3\text{Englsberger, Ott, and Albu-Schaffer 2015.}\)
Equation of motion

\[ \ddot{c} = \omega^2 (c - z) + \ddot{g} \]

Floating-base pendulum

- Floating ZMP (eCMP)
- Exact forward integration
- New feasibility condition
Equation of motion
\[ \ddot{c} = \omega^2 (c - z) + \ddot{g} \]

Feasibility conditions
• Constant: \( \omega > 0 \)
• ZMP support cone: \( z \in \mathcal{Z} \)
GOAL
Nonlinear optimization...

- DMS over FIP model
- Adaptive step timings
- Runs at 30 Hz

... but significant failures

- Model is nonconvex
- Noise and delays in ZMP control / COM estimation
  $\Rightarrow$ jumps in PO map
This communication:

**Constrained LQ regulator**

- Linear EoM + linearized ZMP cones = Quadratic Program
- Runs at 300 Hz, recovers locally from failures

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Next communication:

**Spoiler!**

A *convexly-constrained* model: one global optimum, 1000 Hz

https://scaron.info/research/3d-balance.html

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\(^4\)Caron and Mallein 2017.
CHECK IT OUT!

https://github.com/stephane-caron/dynamic-walking
Floating-base Pendulum

- LTI model for 3D walking
- ZMP support area \(\Rightarrow\) cone

Nonlinear Predictive Control

- Can solve full problem
- Failures (nonconvexity)
- Recovery: constrained LQR
THANK YOU FOR YOUR ATTENTION!
