Biped Stabilization by Linear Feedback of the Variable-Height Inverted Pendulum Model

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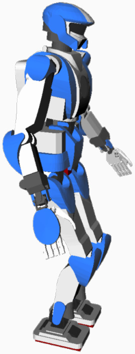
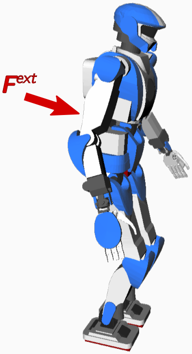
May–August 2020

IEEE Virtual Conference on Robotics and Automation

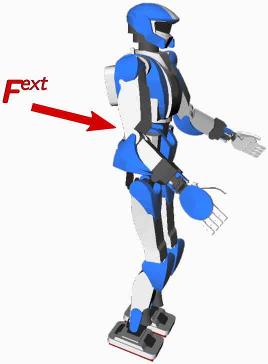
height variation strategy



Rest position



Ankle strategy



Height variation strategy

in this video

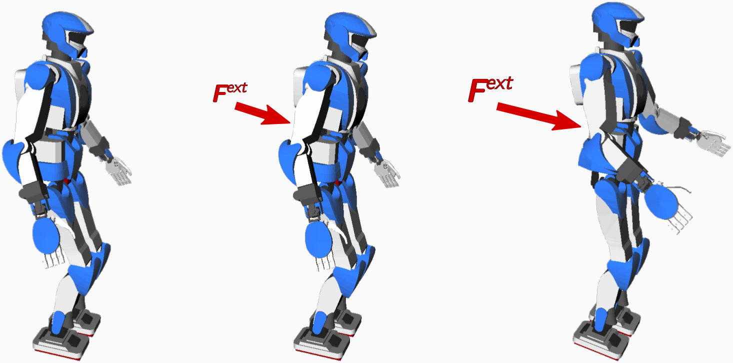
Idea :

- Divergent Component of Motion goes 4D

Techniques :

- Variation dynamics around reference trajectory
- Least-squares pole placement

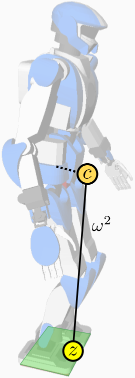
height variation strategy

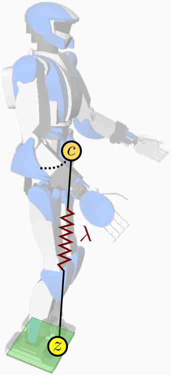


Rest position　　Ankle strategy　　Height variation strategy

1. Twan Koolen, Michael Posa et Russ Tedrake. « Balance control using center of mass height variation : Limitations imposed by unilateral contact ». In : *Humanoids 2016*.

pendulum models





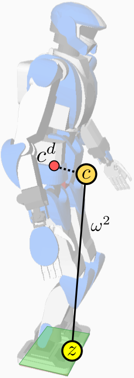
Linear Inverted Pendulum

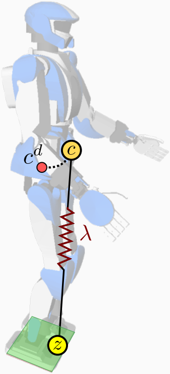
- Model : $\ddot{c} = \omega^2(c - z) + g$
- Inputs : $u = z \in \mathbb{R}^2$

Variable-Height Inverted Pendulum

- Model : $\ddot{c} = \lambda(c - z) + g$
- Inputs : $u = [z \ \lambda] \in \mathbb{R}^3$

balance control





Linear Inverted Pendulum

- State : $\xi = c + \dot{c}/\omega \in \mathbb{R}^3$
- Control : $z = -k(\xi^d - \xi)$

Variable-Height Inverted Pendulum

- State : $[c\ \dot{c}] \in \mathbb{R}^6$
- Control : nonlinear MPC [1]

3D DCM

$$\xi := c + \frac{\dot{c}}{\omega}$$

Divergent dynamics

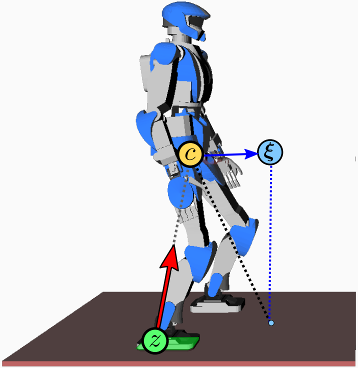
$$\dot{\xi} = \omega(\xi - z)$$

Convergent dynamics

$$\dot{c} = \omega(\xi - c)$$

Viability

Diverges iff $\xi \notin support(z)$



2. Johannes Englsberger, Christian Ott et Alin Albu-Schäffer. « Three-dimensional bipedal walking control based on divergent component of motion ». In : *IEEE T-RO* (2015).

Tracking error : $\Delta\xi := \xi - \xi^d$

Error dynamics

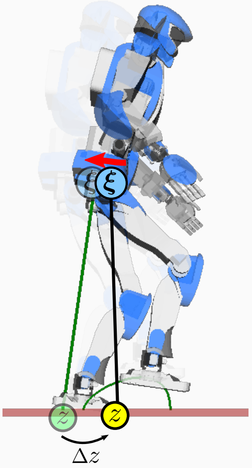
$$\Delta\dot{\xi} = \omega(\Delta\xi - \Delta z)$$

Desired error dynamics

$$\Delta\dot{\xi}^* = -k_p \Delta\xi$$

Derive feedback

$$\Delta z^* = \arg\min_{\Delta z} \|\Delta\xi^* - \Delta\xi\| = -\left[1 + \frac{k_p}{\omega}\right] \Delta\xi$$



3. Mitsuharu Morisawa, Nobuyuki Kita, Shin'ichiro Nakaoka, Kenji Kaneko, Shuuji Kajita et Fumio Kanehiro. « Balance control based on capture point error compensation for biped walking on uneven terrain ». In : *Humanoids 2012*.

Model

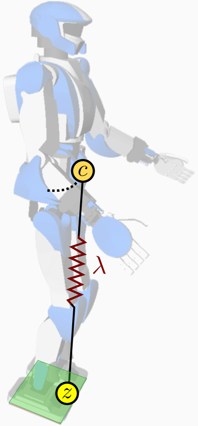
$$\ddot{c} = \lambda(c - z) + g$$

Pre-defining $c_z(t) \to \lambda(t)$ makes system LTV :

Time-varying DCM

$$\xi = c + \frac{\dot{c}}{\omega(t)}$$

with the Riccati equation $\dot{\omega} = \omega^2 - \lambda$.

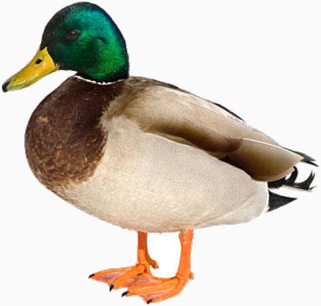


4. Michael A. Hopkins, Dennis W. Hong et Alexander Leonessa. « Humanoid locomotion on uneven terrain using the time-varying Divergent Component of Motion ». In : *Humanoids 2014*.

Cartesian space	*Phase space*
Input	**Input**
z	λ
Divergent dynamics	**Divergent dynamics**
$\dot{\xi} = \omega(\xi - z)$	$\dot{\omega} = \omega^2 - \lambda$
Convergent dynamics	**Convergent dynamics**
$\dot{\zeta} = \omega(z - \zeta)$	$\dot{\gamma} = \lambda - \gamma^2$
Viability	**Viability**
Diverges iff $\xi \notin support(z)$	Diverges iff $\omega^2 \notin [\lambda_{min}, \lambda_{max}]$

5. Stéphane Caron, Adrien Escande, Leonardo Lanari et Bastien Mallein. « Capturability-based Pattern Generation for Walking with Variable Height ». In : *IEEE T-RO* (2020).

interpretation



Divergent component of motion

$$x = [\xi \ \omega] \in \mathbb{R}^4 : \text{Cartesian 3D DCM } \xi + \text{natural frequency } \omega$$

Divergent dynamics

$$\dot{x} = \begin{bmatrix} \dot{\xi} \\ \dot{\omega} \end{bmatrix} = \frac{1}{\omega} \begin{bmatrix} \lambda I_3 & 0 \\ 0 & \omega^2 \end{bmatrix} x - \frac{1}{\omega} \begin{bmatrix} \lambda I_3 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} z \\ \lambda \end{bmatrix} + \frac{1}{\omega} \begin{bmatrix} g \\ 0 \end{bmatrix}$$

Divergent component of motion

$x = [\xi\ \omega] \in \mathbb{R}^4$: Cartesian 3D DCM ξ + natural frequency ω

Divergent dynamics

$$\dot{x} = \begin{bmatrix} \dot{\xi} \\ \dot{\omega} \end{bmatrix} = \frac{1}{\omega} \begin{bmatrix} \lambda I_3 & 0 \\ 0 & \omega^2 \end{bmatrix} x - \frac{1}{\omega} \begin{bmatrix} \lambda I_3 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} z \\ \lambda \end{bmatrix} + \frac{1}{\omega} \begin{bmatrix} g \\ 0 \end{bmatrix}$$

Take its linearized error dynamics (*a.k.a.* variation dynamics) :

Tracking error dynamics

$$\Delta \dot{x} = \frac{1}{\omega^d} \begin{bmatrix} \lambda^d I_3 & -\ddot{c}^d / \omega^d \\ 0 & 2(\omega^d)^2 \end{bmatrix} \Delta x - \frac{1}{\omega^d} \begin{bmatrix} \lambda^d I_3 & (\xi^d - z^d) \\ 0 & \omega^d \end{bmatrix} \begin{bmatrix} \Delta z \\ \Delta \lambda \end{bmatrix}$$

Linear system : $\Delta \dot{x} = A \Delta x + B \Delta u$.

Error dynamics

$$\Delta\dot{x} = A\Delta x + B\Delta u$$

Desired error dynamics

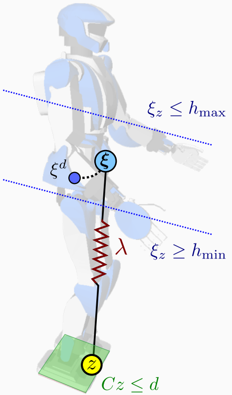
$$\Delta\dot{x}^* = -k_p\Delta x$$

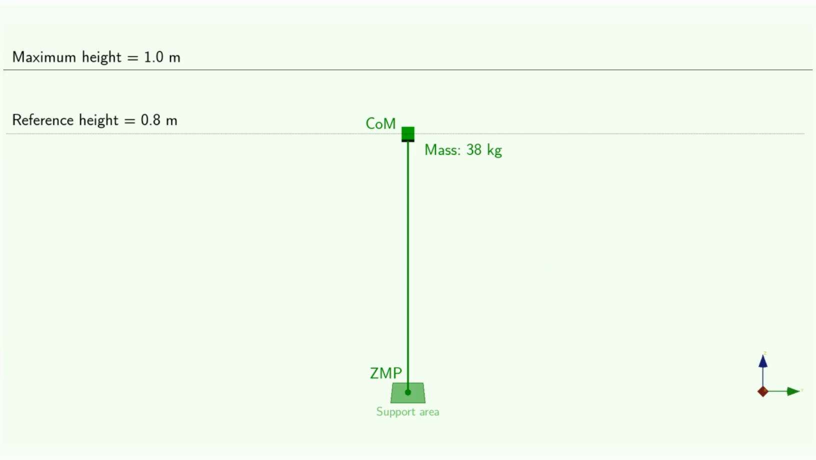
Derive feedback

Minimize : $\|\Delta\dot{x} - \Delta\dot{x}^*\|^2$

Subject to :

- Linearized dynamics : $\Delta\dot{x} = A\Delta x + B\Delta u$
- ZMP support area : $C(z^d + \Delta z) \leq d$
- Reaction force : $\lambda_{min} \leq \lambda^d + \Delta\lambda \leq \lambda_{max}$
- Kinematics : $h_{min} \leq \xi_z^d + \Delta\xi_z \leq h_{max}$

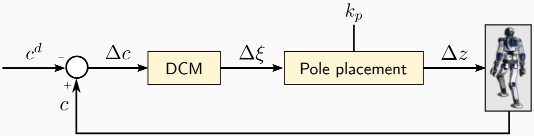




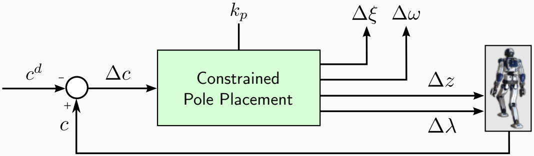
https://github.com/stephane-caron/pymanoid/blob/master/examples/vhip_stabilization.py

controller adjusts the dcm

Previously, the DCM was a measured state :



Now, the DCM is an **output** that can be adjusted :



The controller can vary ω (height) to maintain $\xi \in \mathit{support}(z)$.

force control

LIP tracking

VHIP tracking

https://github.com/stephane-caron/vhip_walking_controller

what have we seen ?

Idea :

- Divergent Component of Motion goes 4D

Techniques :

- Variation dynamics around reference trajectory
- Least-squares pole placement

To go further

- Link with *exponential dichotomies* (Coppel, 1967) : `https://scaron.info/talks/jrl-2019.html`

Thank you for your attention !

[1] Stéphane Caron, Adrien Escande, Leonardo Lanari et Bastien Mallein. « Capturability-based Pattern Generation for Walking with Variable Height ». In : *IEEE T-RO* (2020).

[2] Johannes Englsberger, Christian Ott et Alin Albu-Schäffer. « Three-dimensional bipedal walking control based on divergent component of motion ». In : *IEEE T-RO* (2015).

[3] Michael A. Hopkins, Dennis W. Hong et Alexander Leonessa. « Humanoid locomotion on uneven terrain using the time-varying Divergent Component of Motion ». In : *Humanoids 2014*.

[4] Twan Koolen, Michael Posa et Russ Tedrake. « Balance control using center of mass height variation : Limitations imposed by unilateral contact ». In : *Humanoids 2016*.

[5] Mitsuharu Morisawa, Nobuyuki Kita, Shin'ichiro Nakaoka, Kenji Kaneko, Shuuji Kajita et Fumio Kanehiro. « Balance control based on capture point error compensation for biped walking on uneven terrain ». In : *Humanoids 2012*.