Balance control using both ZMP and COM height variations: A convex boundedness approach

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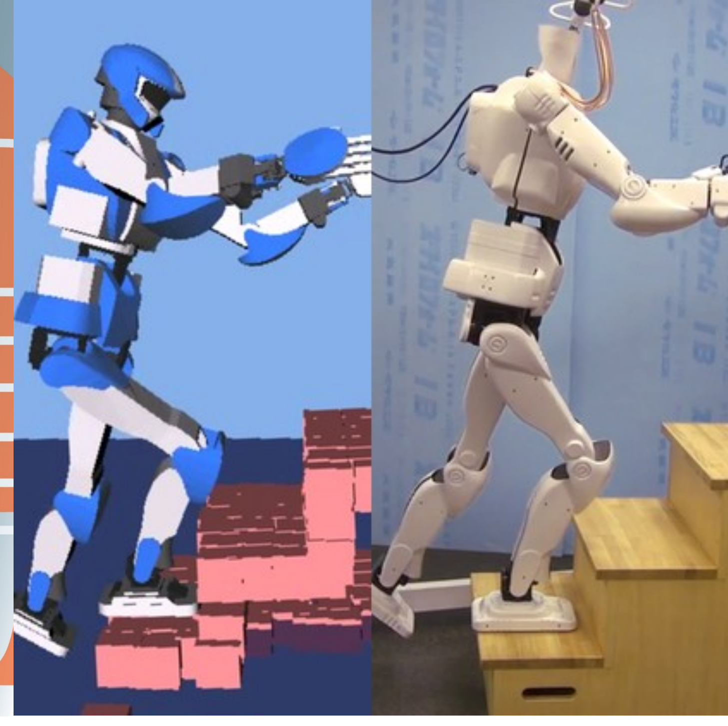


Trajectory-free height variations for 3D bipedal walking

Boundedness condition

- **DCM dynamics:** $\dot{\boldsymbol{\xi}} = \omega(t)\boldsymbol{\xi} + \boldsymbol{g} - \lambda(t)\boldsymbol{r}$

  The solution to this differential equation is:

  $$\boldsymbol{\xi}(t) = e^{\Omega(t)}\left(\boldsymbol{\xi}(0) + \int_0^t e^{-\Omega(\tau)}(\lambda(\tau)\boldsymbol{r}(\tau) - \boldsymbol{g})\mathrm{d}\tau\right)$$

- **Observation:** as $t \to \infty$ the DCM $\boldsymbol{\xi}(t)$ should stay bounded
- Leads us to the **boundedness condition:**

  $$\boldsymbol{\xi}(0) = \int_0^\infty (\lambda(t)\boldsymbol{r}(t) - \boldsymbol{g})\mathrm{d}t$$

  Constraint between the current state (LHS) and *all* future inputs $\lambda(t), \boldsymbol{r}(t)$ of the IPM (RHS)

Problem formulation

- **Change of variable:** $s = e^{-\Omega(t)}$

  So that the boundedness condition becomes:

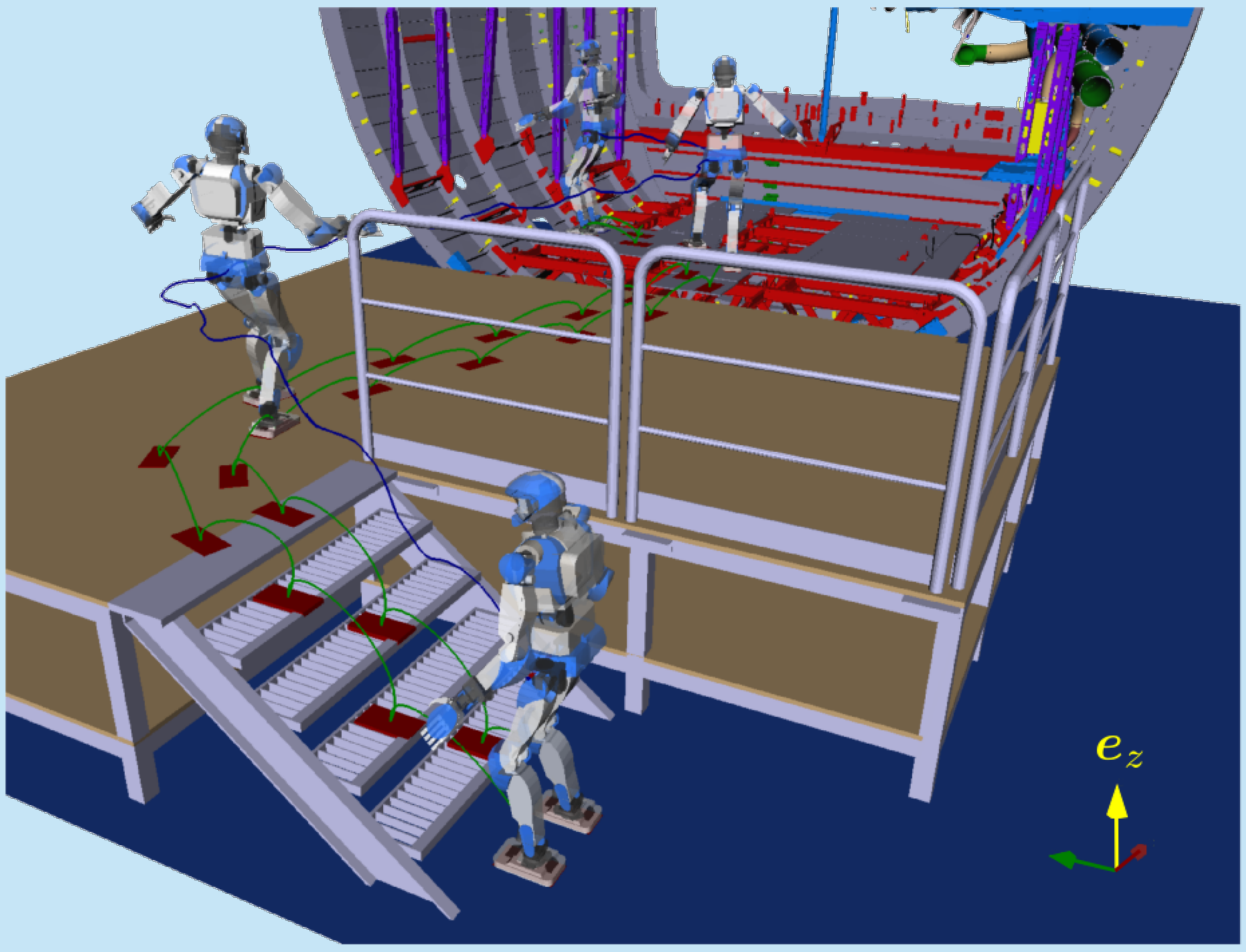
  $$\int_0^1 \boldsymbol{r}^{xy}(s)(s\omega(s))'\mathrm{d}s = \dot{\boldsymbol{c}}_i^{xy} + \omega_i \boldsymbol{c}_i^{xy}$$

  $$g\int_0^1 \frac{1}{\omega(s)}\mathrm{d}s = \dot{\boldsymbol{c}}_i^{z} + \omega_i \boldsymbol{c}_i^{z}$$

- **Optimize** over $\varphi_i = s_i^2\omega(s_i)^2$
- **From** φ^*, derive $\lambda(s), \omega(s), \lambda(t), \omega(t), \boldsymbol{r}(t), \boldsymbol{c}(t), \ldots$

*** Work with A. Escande and L. Lanari:**
"*Capturability-based Analysis, Optimization and Control of 3D Bipedal Walking*", Caron, Escande, Lanari & Mallein, Submitted.
https://hal.archives-ouvertes.fr/hal-01689331

Application to 3D walking*



Walking into an A350 aircraft under construction. Scenario provided by Airbus Group, H2020 EU project COMANOID.

Legend
- CoM trajectory
- Swing-foot trajectories

Optimization problem

$$\underset{\varphi_1,\ldots,\varphi_N}{\text{minimize}} \quad \sum_{j=1}^{N-1}\left[\frac{\varphi_{j+1}-\varphi_j}{\Delta_j} - \frac{\varphi_j - \varphi_{j-1}}{\Delta_{j-1}}\right]^2$$ minimize height variations

$$\text{subject to} \quad \sum_{j=0}^{N-1}\frac{\Delta_j}{\sqrt{\varphi_{j+1}}+\sqrt{\varphi_j}} - \frac{c_i^z}{g}\sqrt{\varphi_N} = \frac{\dot{c}_i^z}{g}$$ boundedness condition

$$\omega_{i,\min}^2 \leq \varphi_N \leq \omega_{i,\max}^2$$ CoP condition

$$\forall j,\ \lambda_{\min}\Delta_j \leq \varphi_{j+1} - \varphi_j \leq \lambda_{\max}\Delta_j$$ unilaterality

$$\varphi_1 = \Delta_0 g / z_f$$ terminal CoM height

https://github.com/stephane-caron/capture-walking/

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