Balance control using both ZMP and COM height variations: A convex boundedness approach

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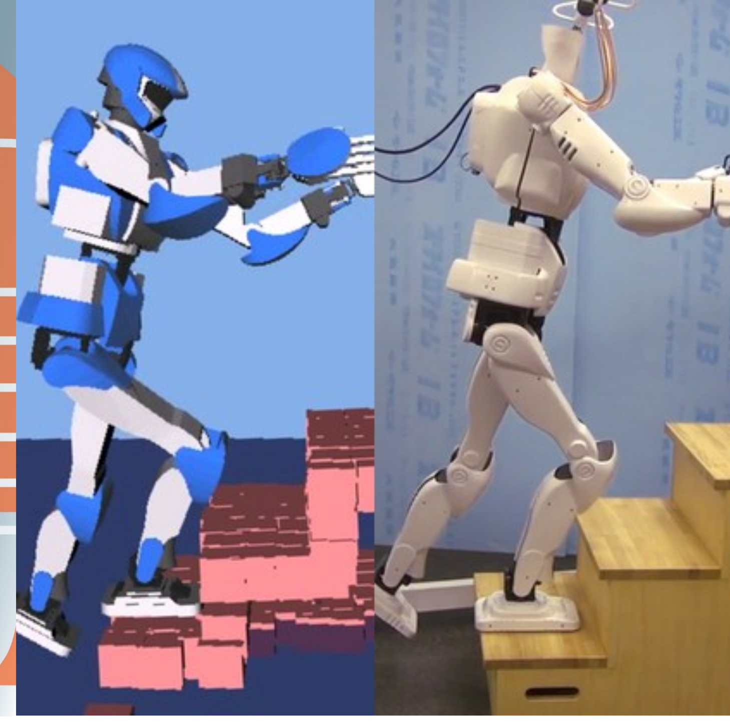


Trajectory-free height variations for 3D bipedal walking

Time-invariant DCM

- **CoM acceleration:** $\ddot{\boldsymbol{c}} = \omega^2(\boldsymbol{c} - \boldsymbol{r})$

  2D walking: $\boldsymbol{r}$ is the ZMP (Zero-tilting Moment Point)
  3D walking: $\boldsymbol{r}$ is the VRP (Virtual Repellent Point)

- **Time-invariant DCM:** $\boldsymbol{\xi} = \boldsymbol{c} + \frac{\dot{\boldsymbol{c}}}{\omega}$

  2D walking: $\boldsymbol{\xi}$ is the Capture Point
  3D walking: $\boldsymbol{\xi}$ is the DCM (Divergent Component of Motion)

- **Decoupled dynamics:** $\begin{cases} \dot{\boldsymbol{\xi}} = \omega(\boldsymbol{\xi} - \boldsymbol{r}) \\ \dot{\boldsymbol{c}} = \omega(\boldsymbol{\xi} - \boldsymbol{c}) \end{cases}$

When possible, setting $\boldsymbol{r} = \boldsymbol{\xi}$ brings the robot to a stop

Time-variant DCM

- **CoM acceleration:** $\ddot{\boldsymbol{c}} = \lambda(t)(\boldsymbol{c} - \boldsymbol{r}) + \boldsymbol{g}$

  3D walking: $\boldsymbol{r}$ is the ZMP (Zero-tilting Moment Point)
  It needs to belong to the foot contact area

- **Time-variant DCM:** $\boldsymbol{\xi} = \dot{\boldsymbol{c}} + \omega(t)\boldsymbol{c}$

  3D walking: $\boldsymbol{\xi}$ can be a position or a velocity
  (velocity version yields simpler formulas here)

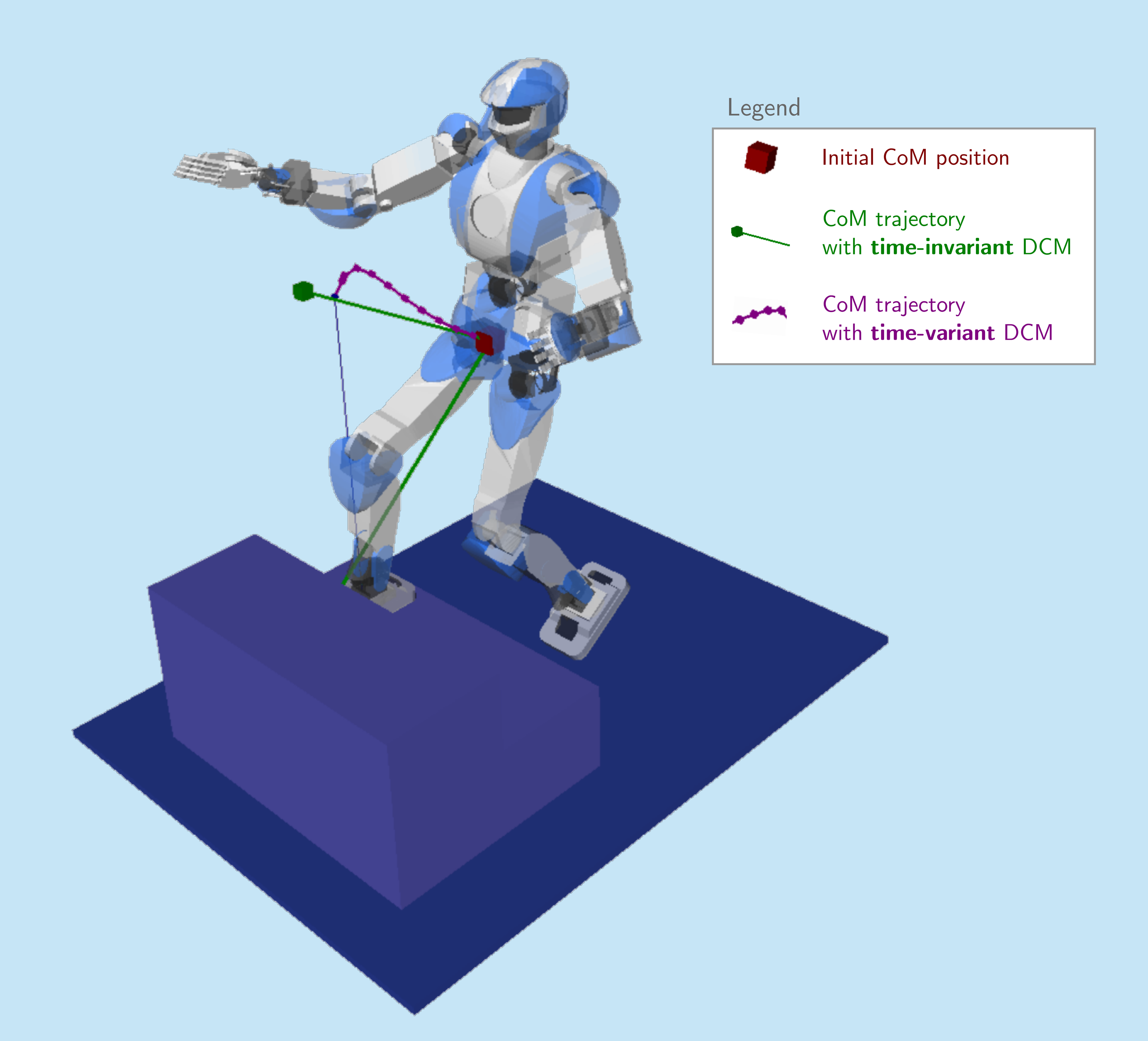
- **DCM dynamics:** $\dot{\boldsymbol{\xi}} = \omega(t)\boldsymbol{\xi} + \boldsymbol{g} - \lambda(t)\boldsymbol{r}$

Subject to the Riccati equation: $\dot{\omega} = \omega^2 - \lambda$

History of these concepts:

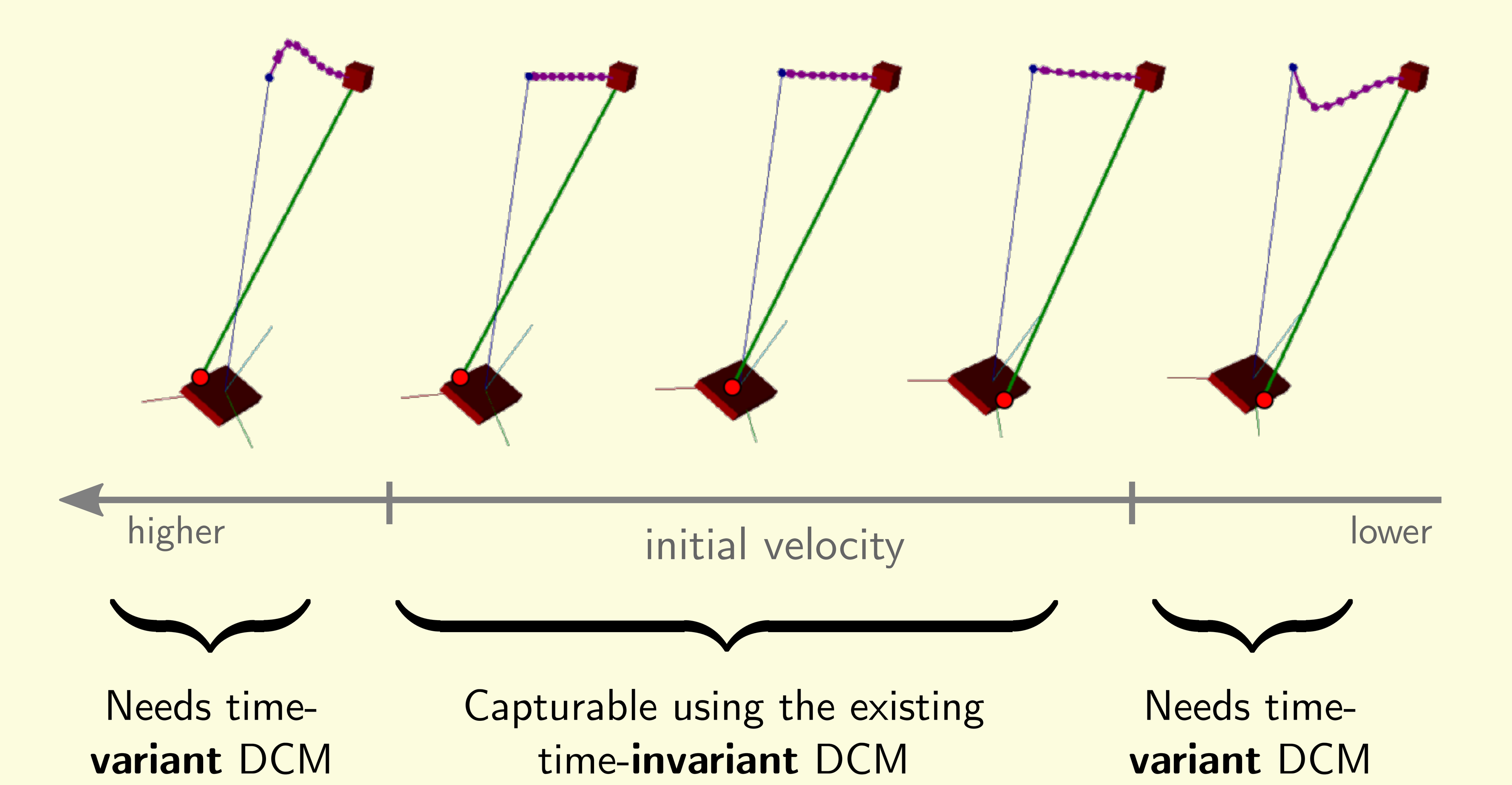
– *Englsberger* et al. (IROS 2013): Time-invariant DCM for 3D walking
– *Lanari* et al. (Hum. 2014): Boundedness condition for 2D walking
– *Hopkins* et al. (Hum. 2014): Time-varying DCM from height trajectory
– *Koolen* et al. (Hum. 2016): Capturability using height variations (2D)

Example of stair climbing



Linear CoM trajectory not feasible because time-invariant DCM is too far. Solving for a time-variant DCM naturally yields additional height variations.

Capturability with height variations



https://github.com/stephane-caron/3d-balance/