STABILITY OF SURFACE CONTACTS: CLOSED-FORM FORMULAE OF THE CONTACT WRENCH CONE FOR RECTANGULAR SUPPORT AREAS

Stéphane Caron[1] Quang-Cuong Pham[2] Yoshihiko Nakamura[1]

[1] Nakamura-Takano Laboratory, UT

[2] CRI Group, NTU

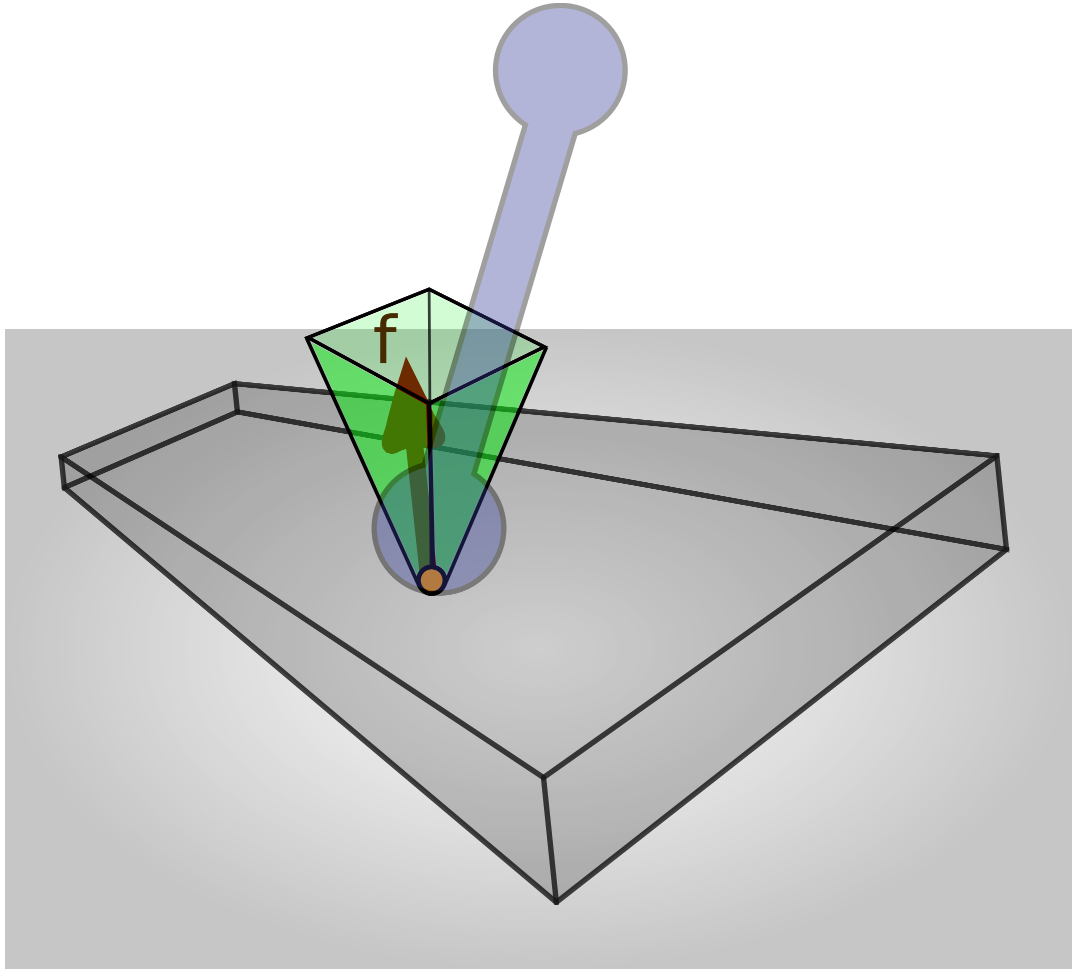
WHAT IS CONTACT STABILITY?

- **Goal:** locomote using environment forces
- **Approach:**
  - Decide contact geometry
  - Maintain that contact (“stability”)

    → *position and force constraints*
  - Generate contact forces

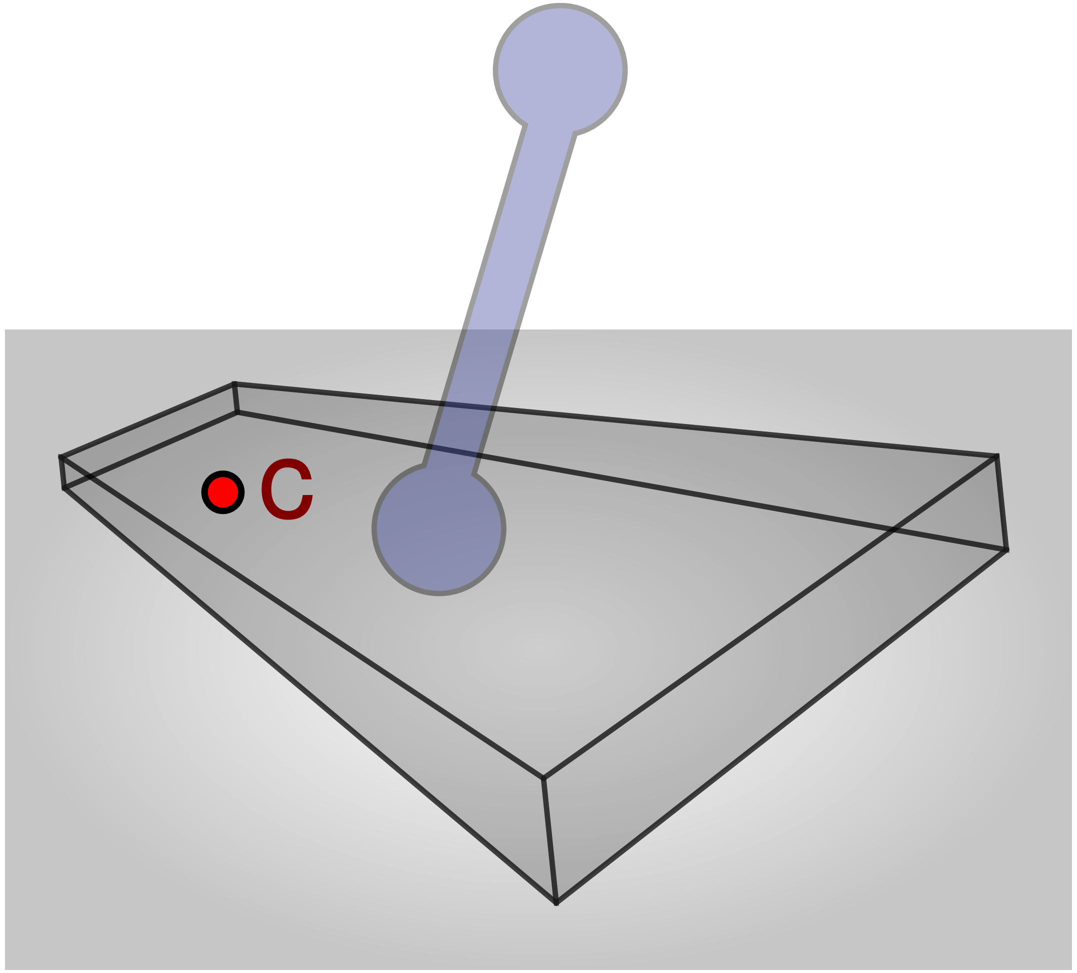
CONTACT CONDITION

- There are 6 DOF of contact:
  - **Three translations**
  - Roll and pitch rotations
  - Yaw rotation
- **Coulomb constraints:**
  - Normal translation: $f_z \geq 0$
  - Surface translation: $|f_t| \leq \mu\, f_z$



COP CONDITION

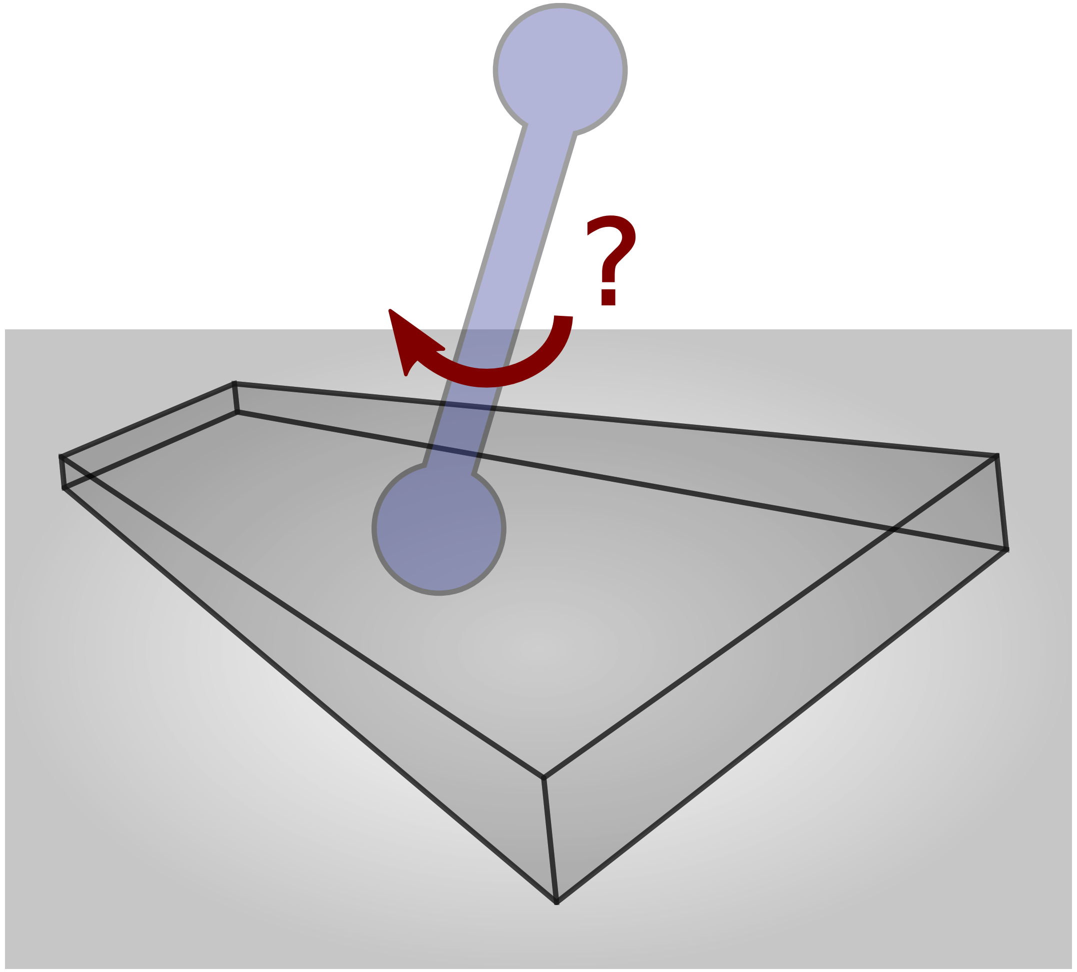
- There are 6 DOF of contact:
  - Three translations
  - **Roll and pitch rotations**
  - Yaw rotations
- **COP constraints:**
  - Roll: $-Y \leq c_y \leq Y$
  - Pitch: $-X \leq c_x \leq X$



COMPLETE CONDITION?

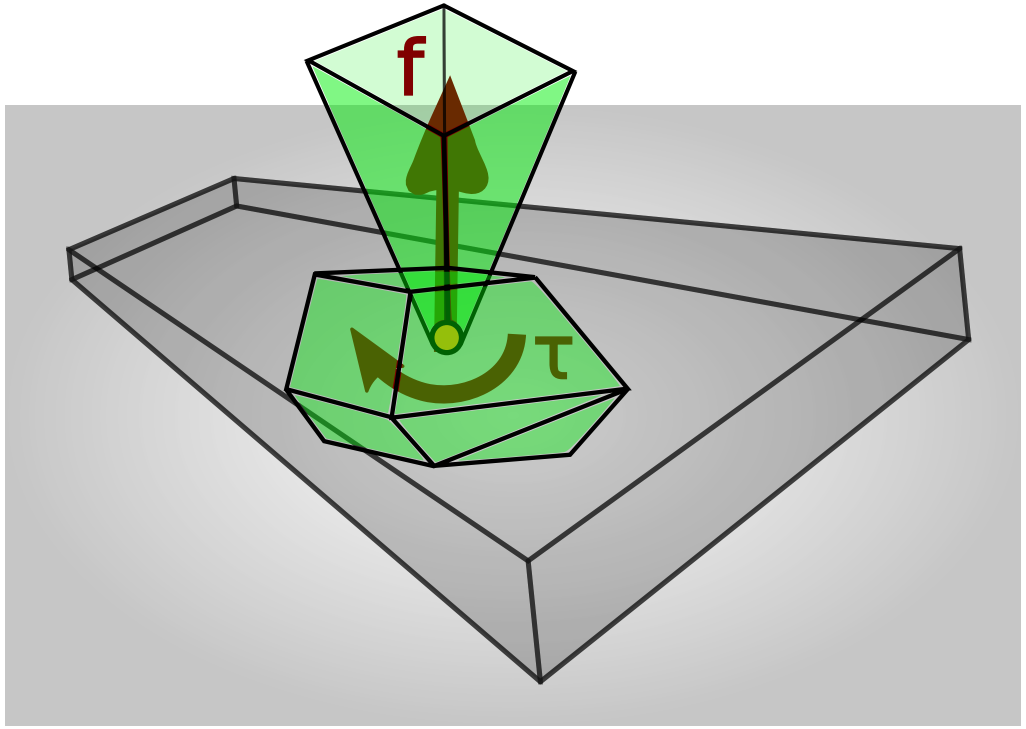
- There are 6 DOF of contact:
  - ~~Three translations~~
  - ~~Roll and pitch rotations~~
  - **Yaw rotation**

⇒ **Contribution of the paper**



SURFACE WRENCH CONE

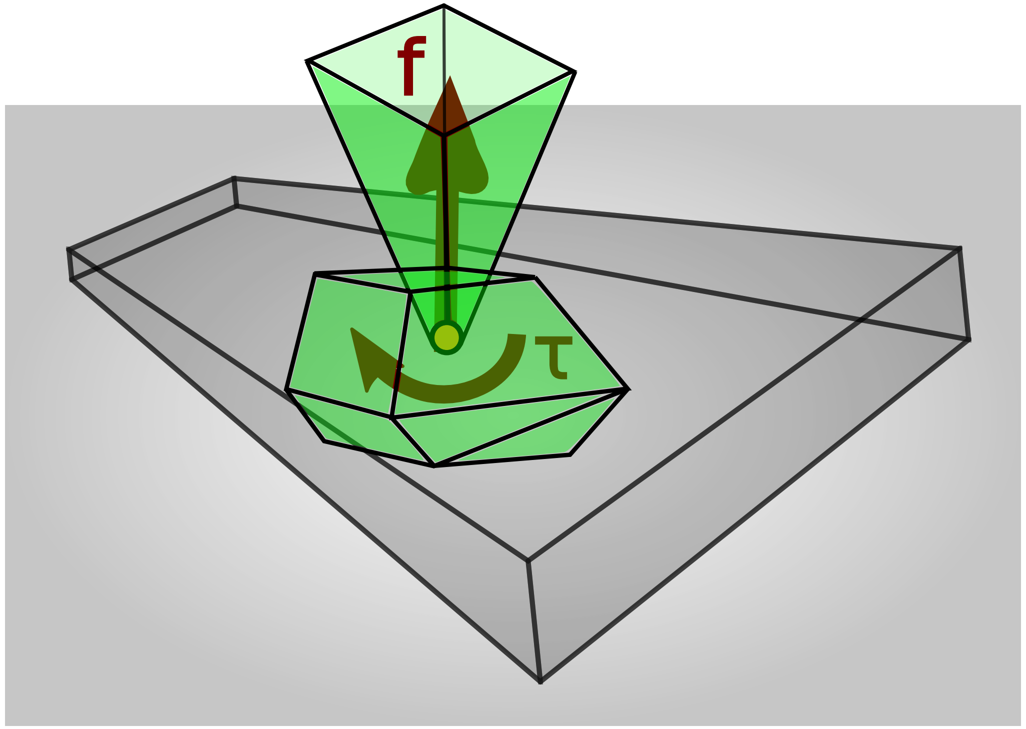
- It constraints all 6 contact DOF
- We have its analytical formula
- Can be used with ≤ 6 DOF,
  e.g. sliding contacts
- Non-redundant representation,
  e.g. directly applicable to TOPP



TOPP: Time-Optimal Path Parameterization

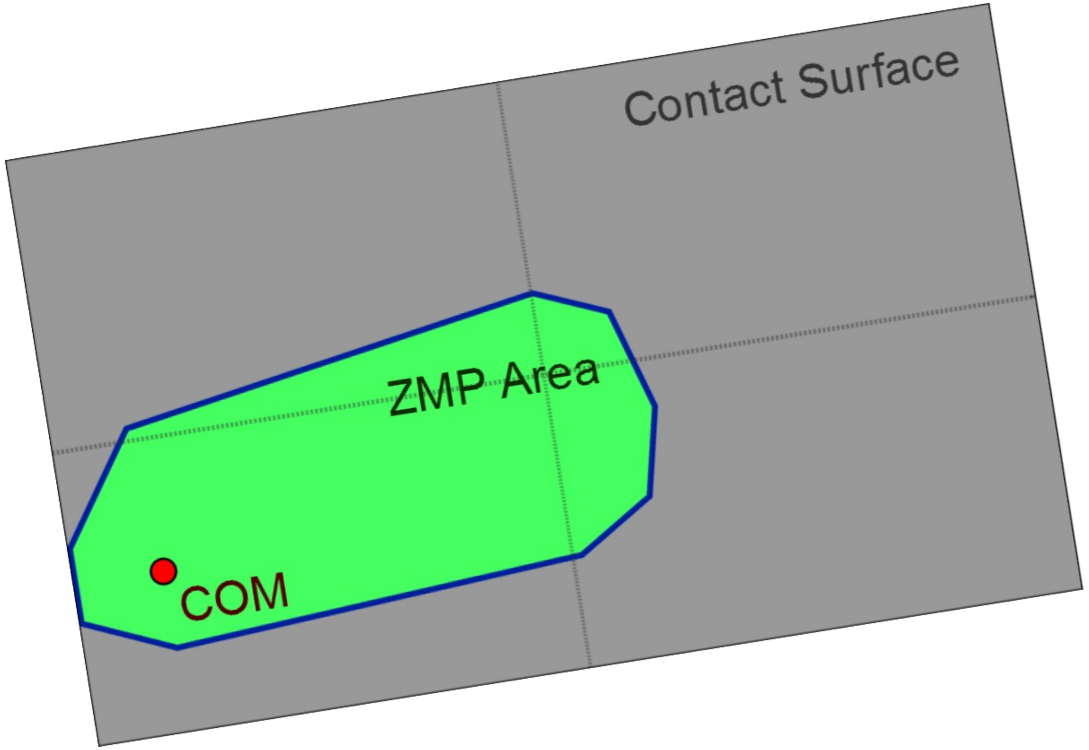
RECTANGULAR SURFACES

- Wrench Cone (16 rows) =
  - Coulomb condition (4 rows)
  - COP condition (4 rows)
  - Yaw condition (8 rows) **(new)**

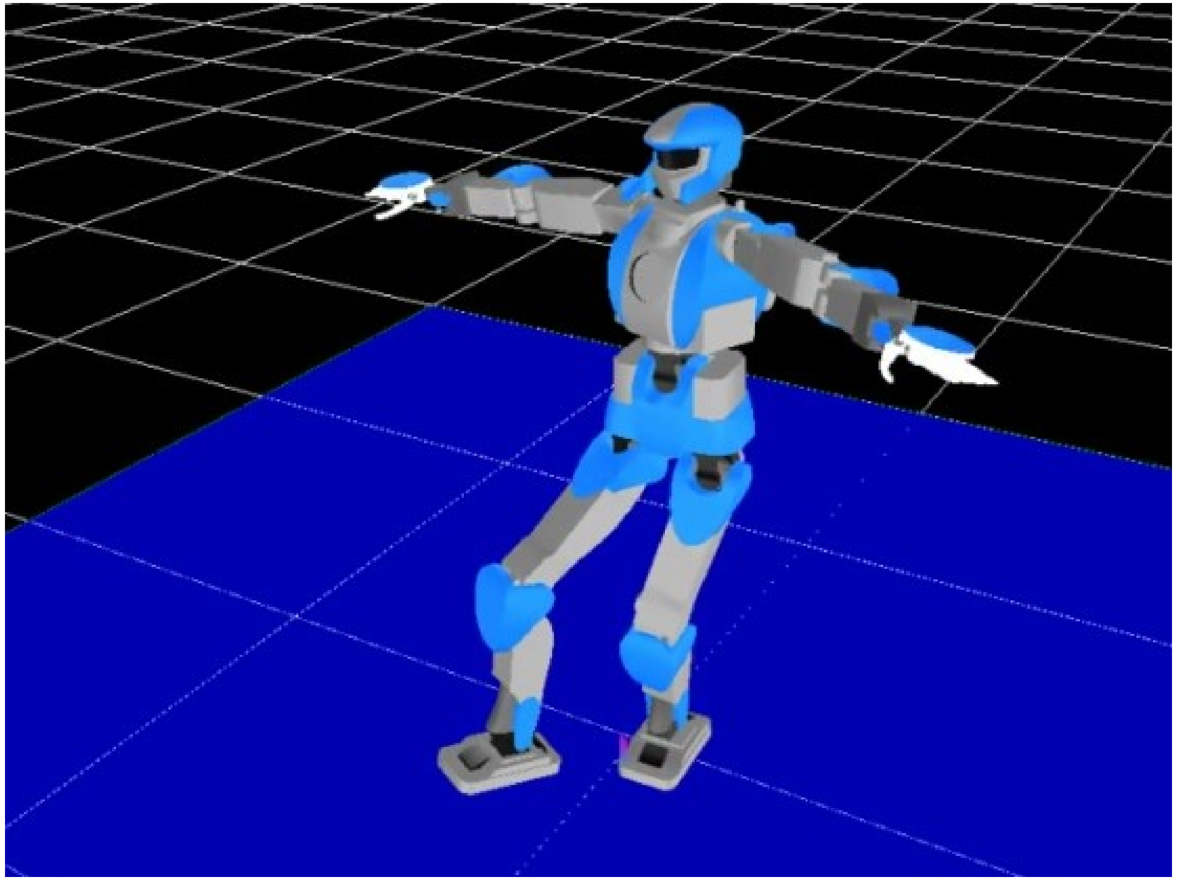


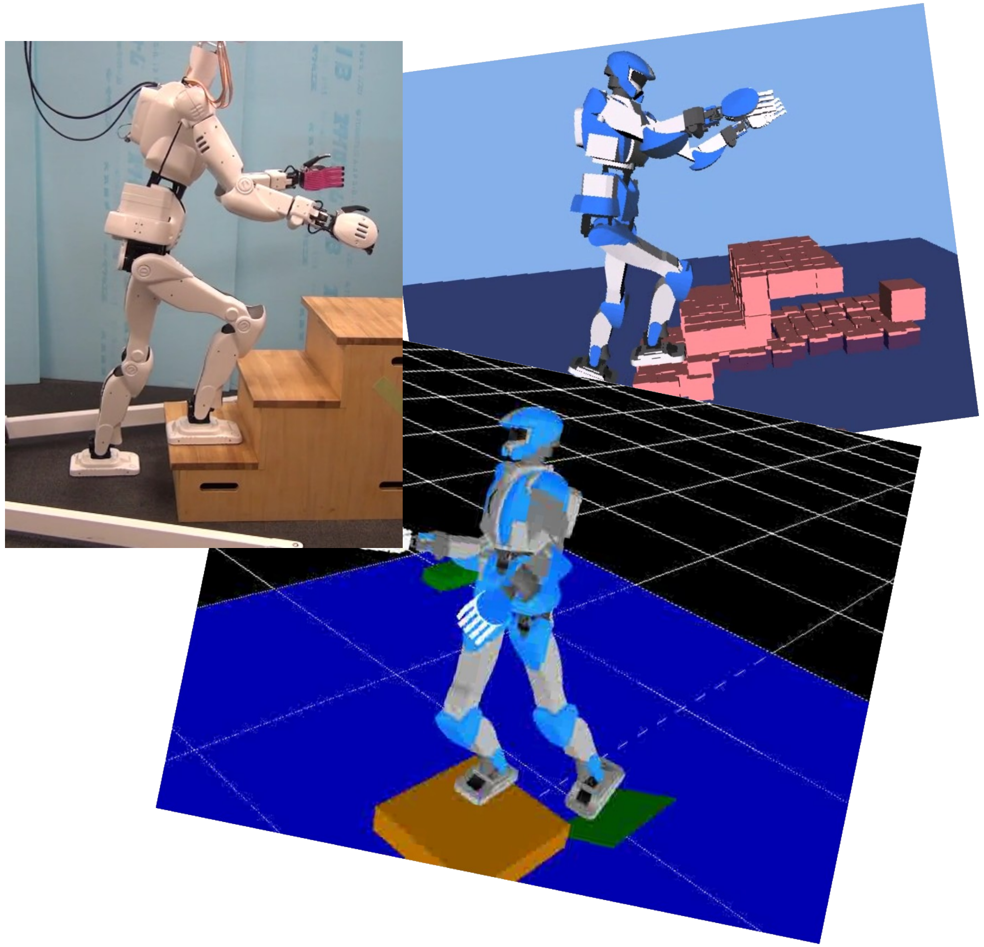
ZMP AREA IN SINGLE CONTACT

- Single-contact: Surface Wrench Cone ⇔ ZMP/COP area
- Area is **smaller** than the foot contact surface
- Figure: $\mu = 0.1$



APPLICATIONS





THANK YOU FOR YOUR ATTENTION :)