Multi-Contact Motion Planning for Humanoids with Cone Duality

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Coulomb Friction (2D example)

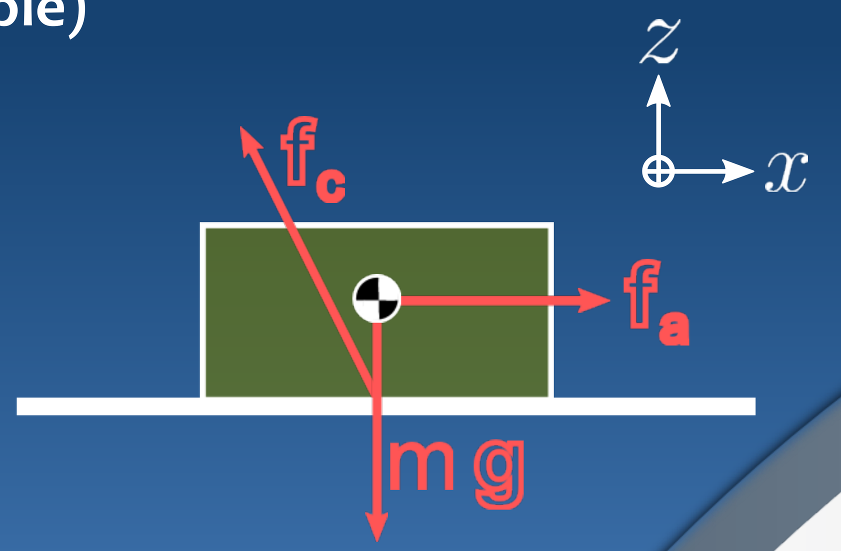
Positional constraint:

$$\ddot{z} = 0$$

Complementarity between accelerations and forces: contact modes

Fixed: $$\begin{cases} \ddot{x} = 0 \\ \|f_c^t\| \leq \mu f_c^z \end{cases}$$

Sliding: $$\begin{cases} \ddot{x} > 0 \\ \|f_c^t\| = -\mu' f_c^z \end{cases}$$



Motivation

Limbed robots need to use contacts to locomote.

Questions

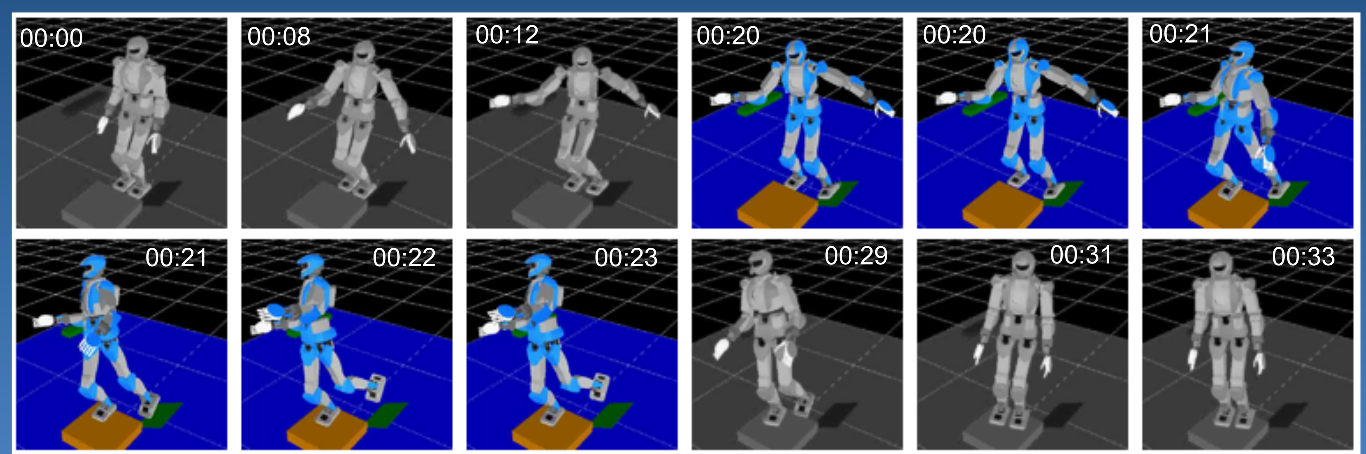
Will a contact hold when executing a motion?
How to make the best use of one contact?
How to change stance (set of contacts)?

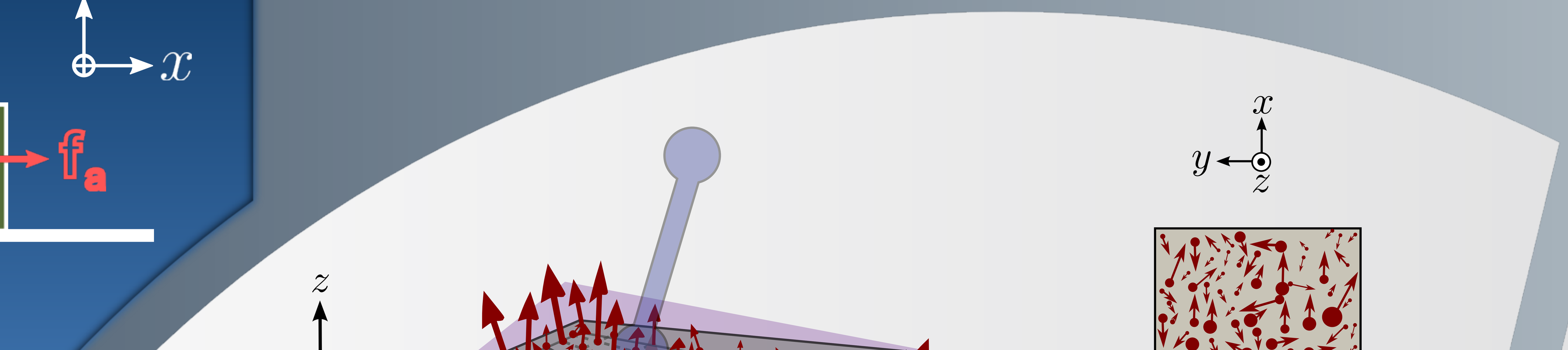
Back to Questions

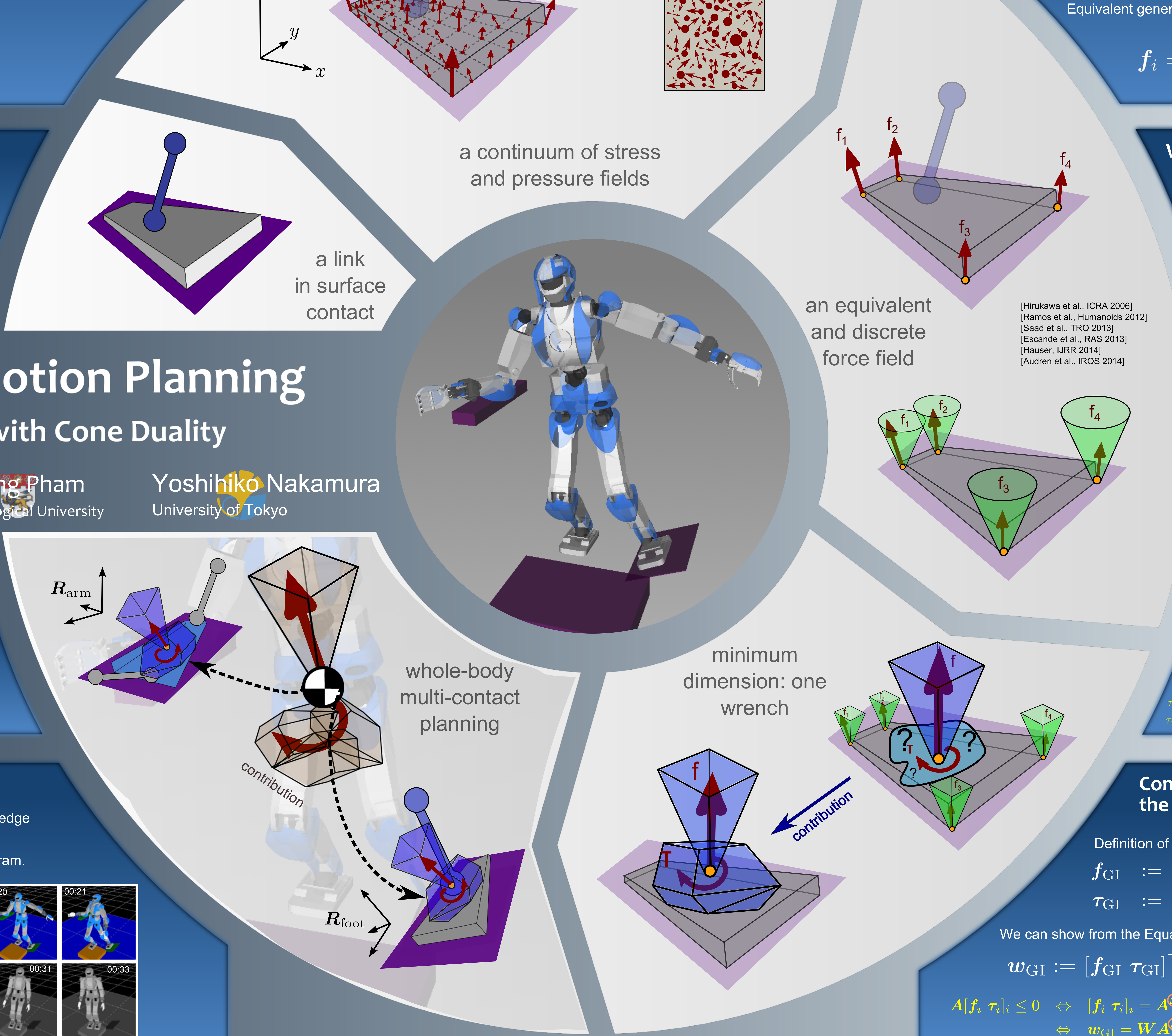
Local: a contact holds by CWC
Whole-body: all contacts hold by GIWC
How to change stance (set of contacts)?
Autonomous discovery of locomotion trajectories?

Simulation Experiment

Stepping on a box using a 25° tilted plane and a 1 m ledge
No solution using statically-stable configurations
Found a dynamic solution with Time-Optimal Path Param.







Reduction to a discrete vector field

Continuous stress-pressure field $\boldsymbol{\nu}(x,y) = (\boldsymbol{\sigma}(x,y), p(x,y))$

Resultant wrench:

$$\boldsymbol{f} = \int_S \boldsymbol{\nu}(x,y)\,\mathrm{d}x\mathrm{d}y$$

$$\boldsymbol{\tau} = \int_S \boldsymbol{c}_{x,y} \wedge \boldsymbol{\nu}(x,y)\,\mathrm{d}x\mathrm{d}y$$

Equivalent generator using barycentring coordinates:

$$\boldsymbol{f}_i = \int_S \alpha_i(x,y)\boldsymbol{\nu}(x,y)\,\mathrm{d}x\mathrm{d}y$$

Wrench Cone for Rectangular Surfaces

[to appear at ICRA 2015]

Input constraints: $\forall$ vertex i,

$$\|\boldsymbol{f}_i^t\|_1 \leq \mu f_i^z$$
$$f_i^z > 0$$

Reduction of the polyedral convex cone (Fourier-Motzkin) yields three conditions:

1. Coulomb friction on the resultant force

$$\|\boldsymbol{f}^t\| \leq \mu f^z$$
$$f^z > 0$$

2. CoP inside the support area

$$|\tau^x| \leq Y f^z$$
$$|\tau^y| \leq X f^z$$

3. A new condition on the resultant yaw torque

$$\tau_{\min}(\boldsymbol{f}, \tau^x, \tau^y) \leq \tau^z \leq \tau_{\max}(\boldsymbol{f}, \tau^x, \tau^y)$$

$$\begin{aligned} \tau_{\min} &:= -\mu(X+Y)f^z + |Yf^x - \mu\tau^x| + |Xf^y - \mu\tau^y|, \\ \tau_{\max} &:= +\mu(X+Y)f^z - |Yf^x + \mu\tau^x| - |Xf^y + \mu\tau^y| \end{aligned}$$

Combining Contacts at the Gravito-Inertial Wrench

Definition of the GI wrench:

$$\begin{aligned} \boldsymbol{f}_{\mathrm{GI}} &:= m(\boldsymbol{g} - \ddot{\boldsymbol{p}}_{\mathrm{CoM}}) \\ \boldsymbol{\tau}_{\mathrm{GI}} &:= \boldsymbol{p}_{\mathrm{CoM}} \times m(\boldsymbol{g} - \ddot{\boldsymbol{p}}_{\mathrm{CoM}}) - \dot{\mathcal{L}} \end{aligned}$$

We can show from the Equation of Motion that:

$$\boldsymbol{w}_{\mathrm{GI}} := [\boldsymbol{f}_{\mathrm{GI}}\ \boldsymbol{\tau}_{\mathrm{GI}}]^\top = \boldsymbol{W}[\boldsymbol{f}_1\ \boldsymbol{\tau}_1\ \ldots\ \boldsymbol{f}_k\ \boldsymbol{\tau}_k]^\top$$

$$\begin{aligned} \boldsymbol{A}[\boldsymbol{f}_i\ \boldsymbol{\tau}_i]_i \leq 0 \ &\Leftrightarrow\ [\boldsymbol{f}_i\ \boldsymbol{\tau}_i]_i = \boldsymbol{A}^{S}[\boldsymbol{z}_i]_i(\boldsymbol{z}_i \geq 0) \\ &\Leftrightarrow\ \boldsymbol{w}_{\mathrm{GI}} = \boldsymbol{W}\boldsymbol{A}^{S}[\boldsymbol{z}_i]_i(\boldsymbol{z}_i \geq 0) \ \Leftrightarrow\ (\boldsymbol{W}\boldsymbol{A}^{S})^{F}\boldsymbol{w}_{\mathrm{GI}} \leq 0 \end{aligned}$$

Cone Duality