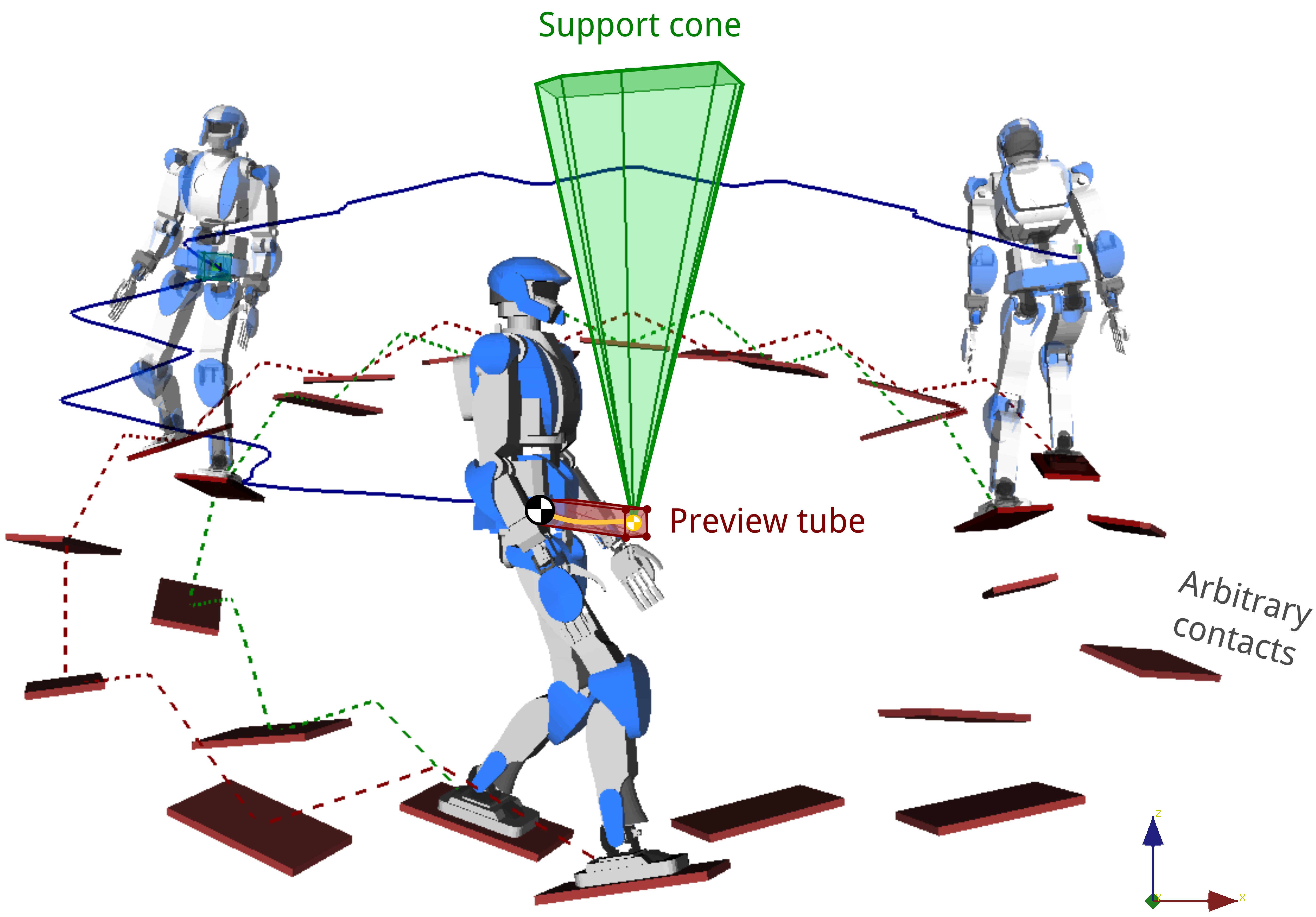
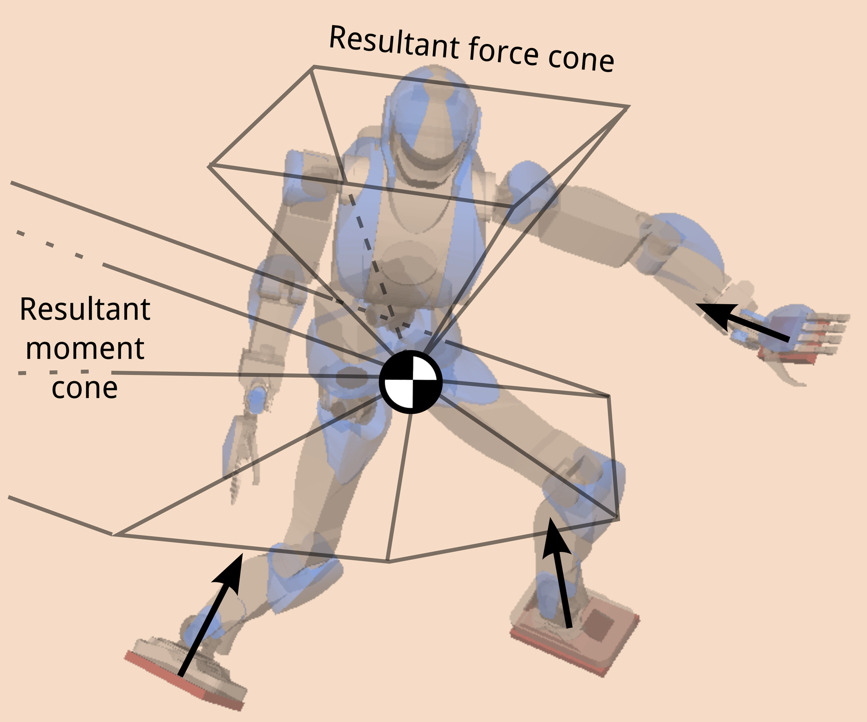
Multi-contact Linear Predictive Control

Stéphane CARON - Abderrahmane KHEDDAR



Multi-contact

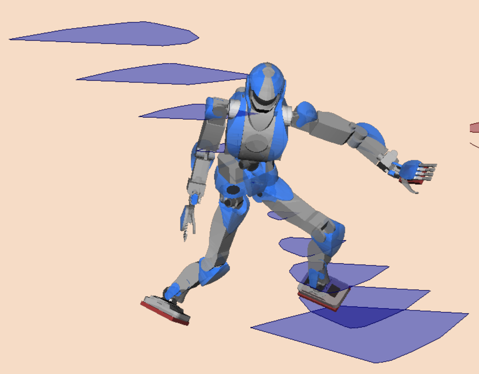
Any set of frictional contacts generates a resultant Contact Wrench Cone:



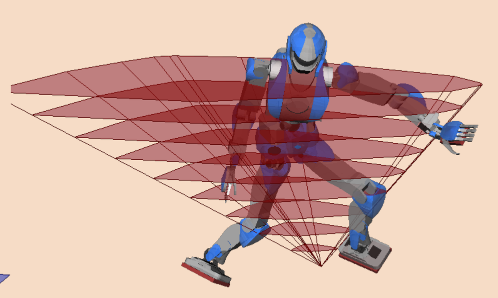
We show how to compute the **reduced wrench cone** for e.g. the Pendulum Mode (constant angular momentum):

$$\dot{L}_G = 0$$

With this new tool, we derive:



Multi-contact ZMP support areas
[Caron *et al.*, TRO 2016]



COM acceleration support cones
[this paper]

Predictive

We preview future COM trajectories with discretized dynamics:

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{u}(k)$$

with $\boldsymbol{x}(k)$: position and velocity of the COM

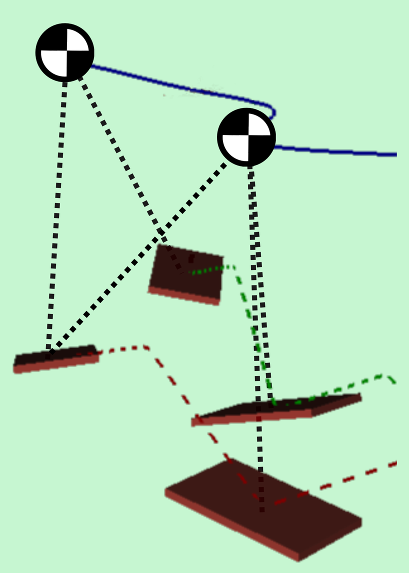
$\boldsymbol{u}(k)$: acceleration of the COM

By Euler integration, future states are given by:

$$\boldsymbol{x}(k) = \boldsymbol{\Phi}\boldsymbol{x}(0) + \boldsymbol{\Psi}\boldsymbol{u}(0 \dots k-1)$$

But **contact stability constraints** are **bilinear**:

$$\boldsymbol{x}(k)^\top \boldsymbol{C}\boldsymbol{u}(k) + \boldsymbol{d}\boldsymbol{u}(k) + \boldsymbol{e} \leq \boldsymbol{0}$$



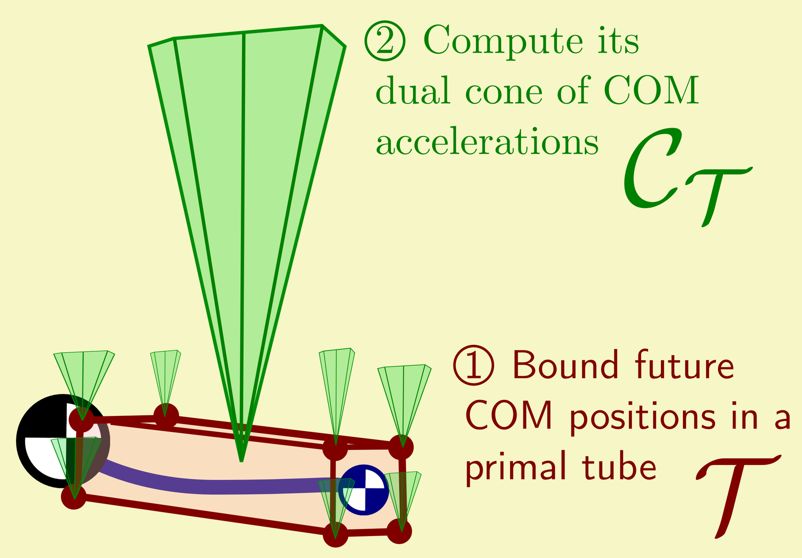
Linear

We show a general method to **linearize** bilinear inequalities:

① Restrict states to lie within pre-defined boundaries

② Propagate these boundaries into bilinear inequalities

Price to pay:
conservative approximation.

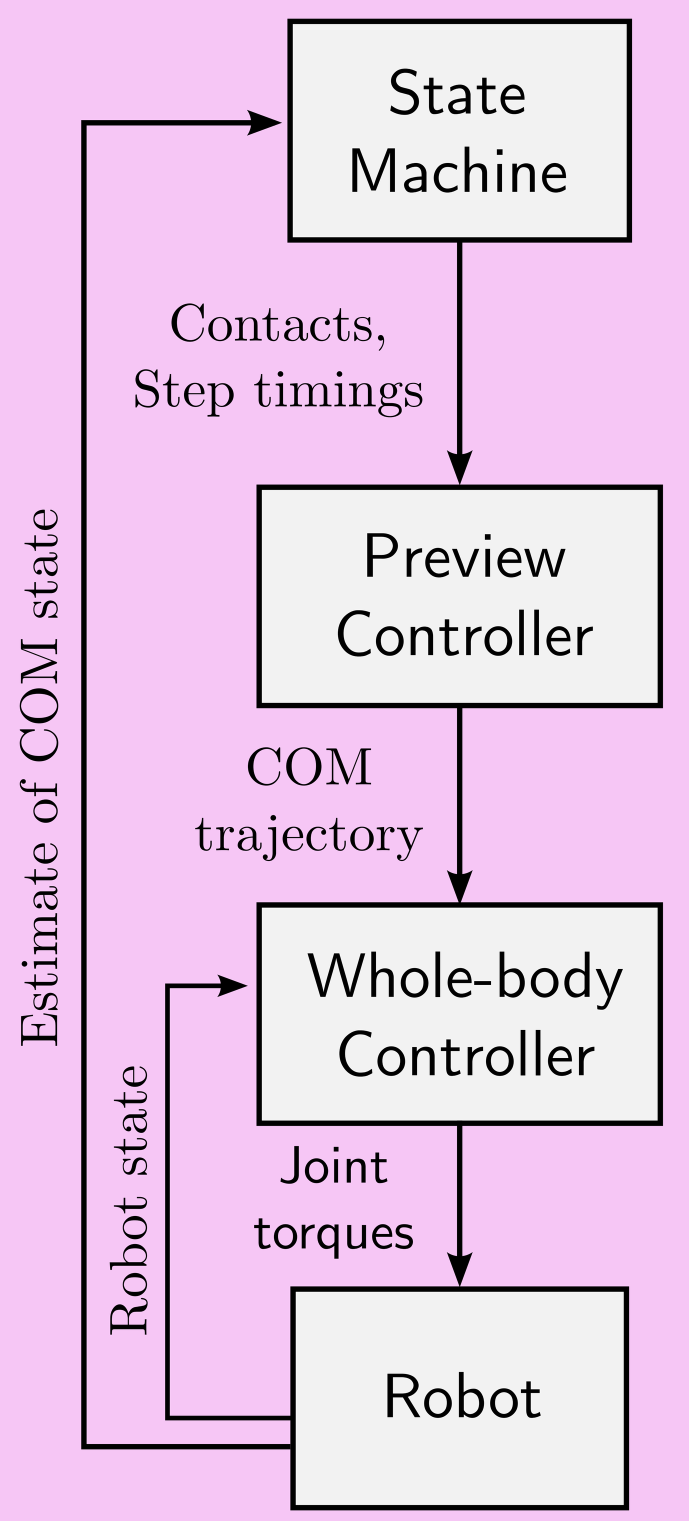


The predictive control problem becomes a **Quadratic Program** (QP):

$$\min_{\boldsymbol{u}(0 \dots N)} \|\boldsymbol{x}(N) - \boldsymbol{x}_{goal}\|^2 + \epsilon \|\boldsymbol{u}(0 \dots N)\|^2$$

$$\text{s.t.} \forall k, \quad \underbrace{\boldsymbol{P}_\mathcal{T}\boldsymbol{x}(k) \leq \boldsymbol{0}}_{\text{primal tube}}, \quad \underbrace{\boldsymbol{I}_\mathcal{T}\boldsymbol{u}(k) \leq \boldsymbol{0}}_{\text{dual cone}}$$

Control



Computation time: **3 ± 2 ms** for the whole control loop

Function	Output	Time (ms)
Double description of $\mathcal{T}$ (SS and DS)	$\boldsymbol{P}_\mathcal{T}$	0.3 ± 0.1
H-representation of $\mathcal{C}_\mathcal{T}$ (SS and DS)	$\boldsymbol{I}^0_\mathcal{T}$	0.2 ± 0.1
Convex hull of $\mathcal{C}_\mathcal{T}$ (SS and DS)	$\boldsymbol{I}_\mathcal{T}$	2.4 ± 1.2
Other matrix operations	–	0.4 ± 0.4
Solving final QP ($N = 10$)	U	0.2 ± 0.1
Total		3.5 ± 2.1

https://github.com/stephane-caron/3d-mpc/